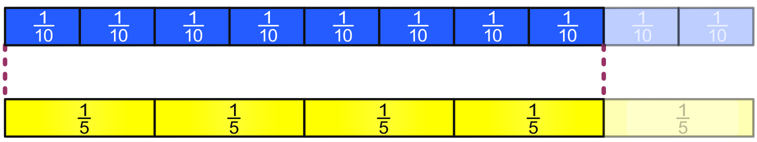
Simplifying Fractions

To enlarge a fraction, we multiply the numerator and the denominator of this fraction by the same natural number (except 0). This gives us a fraction of equal size. The “reverse” process is also possible.
If the numerator and the denominator of a fraction have a common factor, we can divide both by that factor. This gives us a fraction of equal size.

Let’s consider the fraction $\mathbf{\frac{8}{10}}$. As we can see, there is a fraction that is equal in size to $\mathbf{\frac{8}{10}}$, but with a smaller numerator and denominator, namely $\mathbf{\frac{4}{5}}$.



If we divide the denominator 10 by 2, we get 5. $\mathbf{\frac{1}{5}}$ is twice as large as $\mathbf{\frac{1}{10}}$.
To obtain an equal-sized fraction, we therefore only need half as many parts. So we also divide the numerator 8 by 2 and get 4. We have:

$$\frac{8}{10} = \frac{8 \div 2}{10 \div 2} = \frac{4}{5}\ .$$

We can express this connection somewhat imprecisely, but very briefly, as follows:

The bigger the parts, the fewer in number.

The process of dividing the numerator and the denominator of a fraction by a common factor of numerator and denominator is called **simplifying**. Simplifying a fraction results in a fraction of equal size.

> If the numerator and the denominator of a fraction are divided by a common factor, a fraction of equal size results. This process is called **simplifying**.

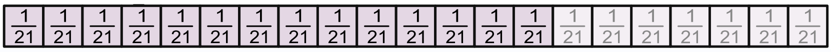
We can also express this connection very briefly:

$$\frac{a}{b} = \frac{a \div n}{b \div n}$$

Here, $\mathbf{\frac{a}{b}}$ stands for any fraction and **n** stands for any common factor of **a** and **b**. To avoid nonsense, we will from now on divide only by such common factors **n** that are greater than **1**.

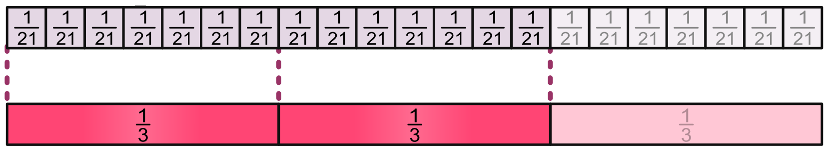
Example 1

We want to simplify the fraction $\frac{14}{21}$ by 7.



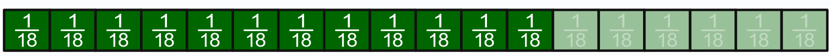
If we divide the denominator 21 by 7, we get 3. $\frac{1}{3}$ is seven times as large as $\frac{1}{21}$.
To get a fraction of equal size, we therefore only need one seventh as many parts.
If we divide the numerator 14 by 7, we get 2.

$$\frac{14}{21} = \frac{14 \div 7}{21 \div 7} = \frac{2}{3}$$



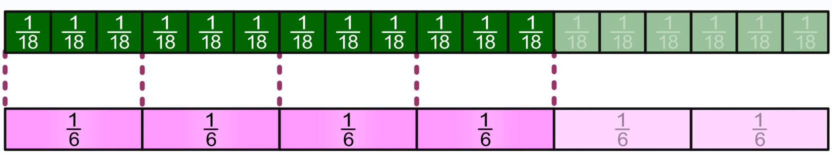
Example 2

We want to simplify the fraction $\frac{12}{18}$ by 3.



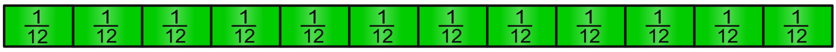
If we divide the denominator 18 by 3, we get 6. $\frac{1}{6}$ is three times as large as $\frac{1}{18}$.
To get a fraction of equal size, we therefore only need one third as many parts.
If we divide the numerator 12 by 3, we get 4.

$$\frac{12}{18} = \frac{12 \div 3}{18 \div 3} = \frac{4}{6}\,.$$

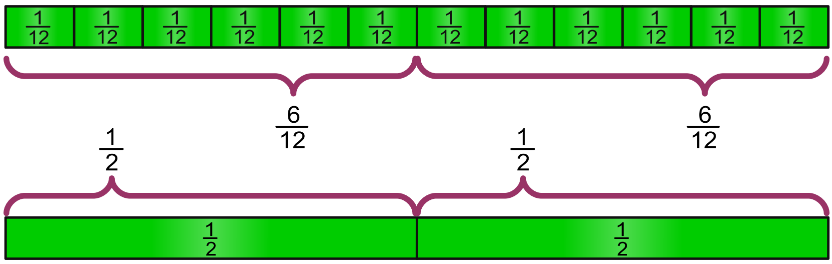


Simplifying by all divisors of the denominator

For every divisor of a denominator, there are fractions that we can simplify by this divisor. Let's look at an example: Here we have 12 twelfths.

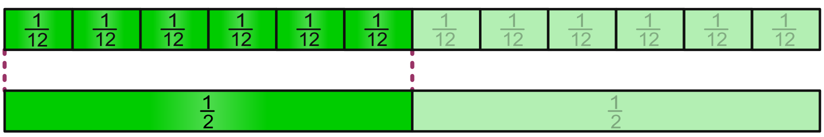


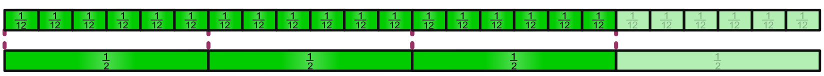
Since 12 is divisible by 6, we can group 6 twelfths into *one* part each. On this fraction strip, there are $12 \div 6 = 2$ such parts. So these are halves.



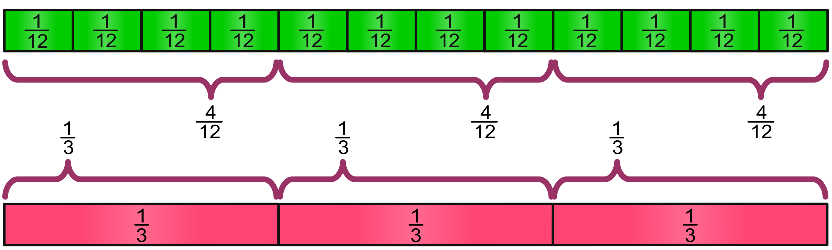
If a fraction has the denominator 12 and a numerator that is divisible by 6, we can simplify the fraction by 6. For example:

$$\frac{6}{12} = \frac{6 \div 6}{12 \div 6} = \frac{1}{2} \text{ and } \frac{18}{12} = \frac{18 \div 6}{12 \div 6} = \frac{3}{2}.$$



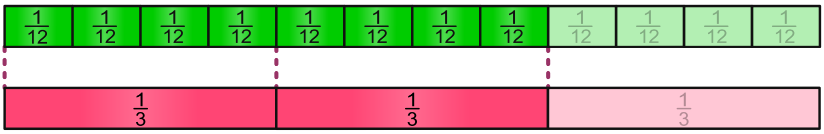


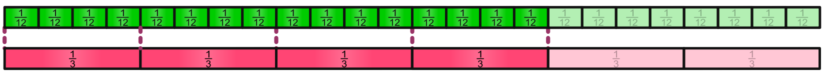
12 is also divisible by 4. Therefore, we can divide the 12 twelfths into groups of four. On one whole, we then have 3 equal parts. Each of these parts is thus equal to $\mathbf{\frac{1}{3}}$. So: $\mathbf{\frac{4}{12} = \frac{1}{3}}$.



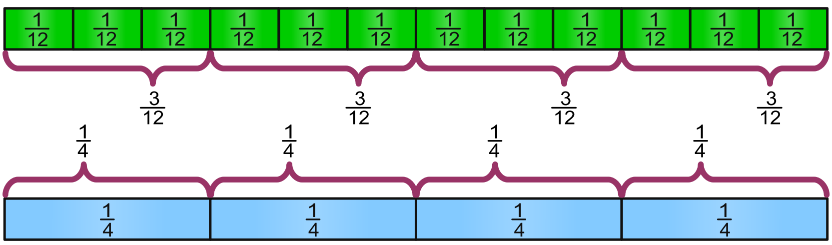
If a fraction has the denominator 12 and a numerator that is divisible by 4, we can simplify the fraction by 4. For example:

$$\frac{8}{12} = \frac{8 \div 4}{12 \div 4} = \frac{2}{3} \text{ and } \frac{16}{12} = \frac{16 \div 4}{12 \div 4} = \frac{4}{3}.$$





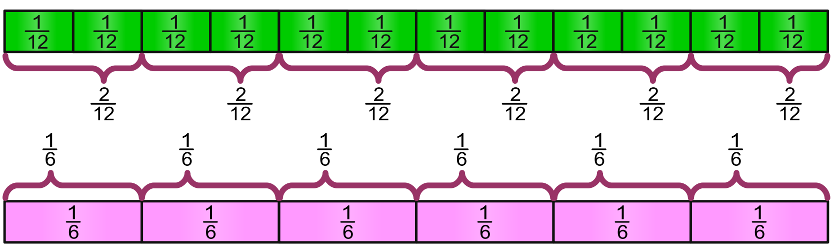
12 is also divisible by 3. Therefore, we can combine 3 twelfths into *one* part and obtain fourths.



If a fraction has the denominator 12 and a numerator that is divisible by 3, we can simplify the fraction by 3. For example:

$$\frac{6}{12} = \frac{6 \div 3}{12 \div 3} = \frac{2}{4} \text{ and } \frac{9}{12} = \frac{9 \div 3}{12 \div 3} = \frac{3}{4}.$$

12 is also divisible by 2. Therefore, we can combine 2 twelfths into *one* part and obtain sixths.



If a fraction has the denominator 12 and a numerator that is divisible by 2, we can simplify the fraction by 2. For example:

$$\frac{6}{12} = \frac{6 \div 2}{12 \div 2} = \frac{3}{6} \text{ and } \frac{28}{12} = \frac{28 \div 2}{12 \div 2} = \frac{14}{6}.$$

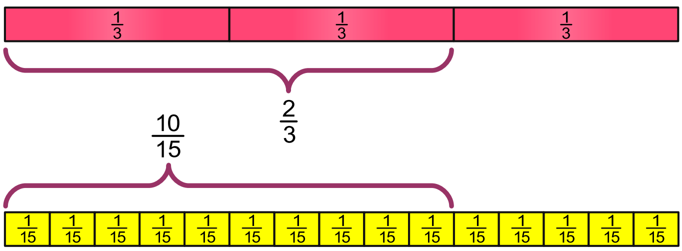
Simplifying as the inverse of expanding

We can also think of simplifying as the inverse of expanding. Expanding means, for example: We divide each third of $\frac{2}{3}$ into 5 equal parts. Each of these parts then has one fifth the size of a third. Therefore, we need 5 times as many parts for a fraction of equal size.

So: $\frac{2}{3} = \frac{2 \times 5}{3 \times 5} = \frac{10}{15}$.

Now we can go the other way and group every 5 of the 10 fifteenths into larger parts. These parts are then 5 times as large as the previous ones, and therefore we only need $10 \div 5 = 2$ parts for a fraction of equal size.

So: $\frac{10}{15} = \frac{10 \div 5}{15 \div 5} = \frac{2}{3}$.



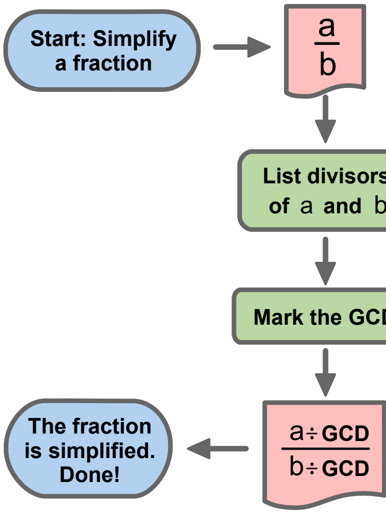
Simplifying a Fraction

Calculating with simplified fractions is usually easier than calculating with unsimplified fractions. We usually simplify a fraction using the greatest common divisor of the numerator and the denominator.
If we are given a fraction, we can proceed as follows:

1. Write down all divisors of the numerator.
2. Write down all divisors of the denominator.
3. Mark the greatest common divisor **GCD** of numerator and denominator.
4. Divide numerator and denominator by the greatest common divisor **GCD**.

From now on, whenever we talk about simplifying a fraction, we always mean simplifying a fraction using the **GCD**.

We can also represent this procedure as a flowchart.



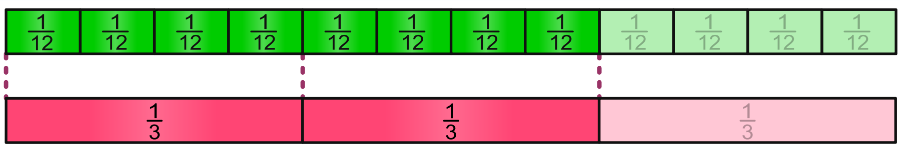
Let's look at some examples:

Example 1

The fraction $\frac{8}{12}$ has numerator 8 and denominator 12.
The divisors of 8 are: $1; 2; 4; 8$.
The divisors of 12 are: $1; 2; 3; 4; 6; 12$.
We have marked the **GCD** in red.
Now we divide the numerator and denominator by the GCD.

$$\frac{8}{12} = \frac{8 \div 4}{12 \div 4} = \frac{2}{3}$$

So if we simplify $\frac{8}{12}$, we get $\frac{2}{3}$.

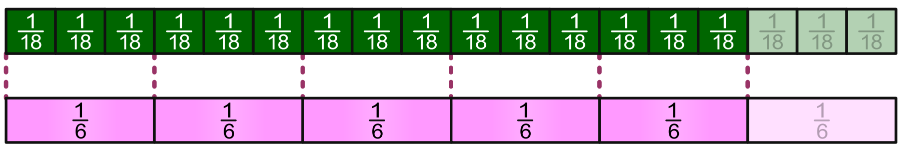


Example 2

The fraction $\frac{15}{18}$ has numerator 15 and denominator 18.
The divisors of 15 are: 1; 3; 5; 15.
The divisors of 18 are: 1; 2; 3; 6; 9; 18.
We have marked the **GCD** in red.
Now we divide the numerator and denominator by the GCD.

$$\frac{15}{18} = \frac{15 \div 3}{18 \div 3} = \frac{5}{6}$$

So if we simplify $\frac{15}{18}$, we get $\frac{5}{6}$.

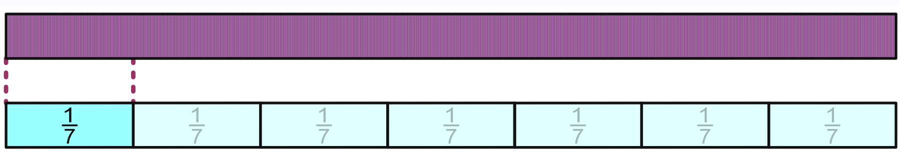


Example 3

The fraction $\frac{108}{756}$ has numerator 108 and denominator 756.
The divisors of 108 are: 1; 2; 3; 4; 6; 9; 12; 18; 27; 36; 54; 108.
The divisors of 756 are: 1; 2; 3; 4; 6; 7; 9; 12; 14; 18; 21; 27; 28; 36; 42; 54; 63; 84; 108; 126; 189; 252; 378; 756.
We have marked the **GCD** in red.
Now we divide numerator and denominator by the GCD.

$$\frac{108}{756} = \frac{108 : 108}{756 : 108} = \frac{1}{7}$$

So, if we reduce the fraction $\frac{108}{756}$, we get $\frac{1}{7}$.



Example 4

The fraction $\mathbf{\frac{40}{81}}$ has numerator 40 and denominator 81.
The divisors of 40 are: 1; 2; 4; 5; 10; 20; 40.
The divisors of 81 are: 1; 3; 9; 27; 81.
We have marked the **GCD** in red.
Since the greatest common divisor of numerator and denominator is 1, this fraction cannot be reduced in a meaningful way.