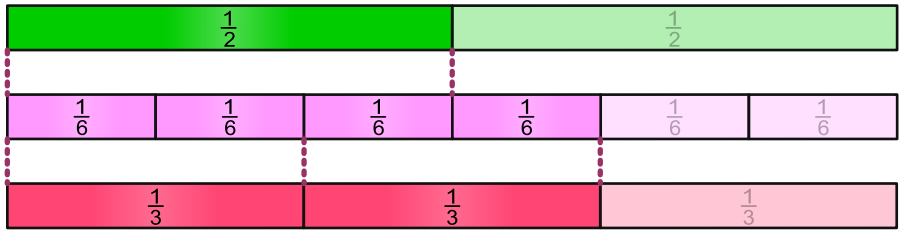
Finding the Least Common Denominator

Making Fractions Have the Same Denominator

We can compare the sizes of fractions especially well, or add and subtract them more easily, when they have the same denominator.
If two fractions have different denominators, we can rewrite them as equivalent fractions with the same denominator.
Let's look at this idea using fraction strips:

We have the fractions $\frac{1}{2}$ and $\frac{2}{3}$. We want to rewrite both fractions as equivalent fractions with the same denominator. Sixths are a good option for this.



We find an equivalent fraction for $\frac{1}{2}$ by multiplying the numerator and the denominator by 3:

$$\frac{1}{2} = \frac{1 \times 3}{2 \times 3} = \frac{3}{6}$$

We find an equivalent fraction for $\frac{2}{3}$ by multiplying the numerator and the denominator by 3:

$$\frac{2}{3} = \frac{2 \times 2}{3 \times 2} = \frac{4}{6}$$

Both resulting fractions now have the same denominator 6.

Vocabulary

The process of changing fractions so that they have the same denominator is called:

- **finding a common denominator**
- **rewriting with a common denominator**
- **making the denominators the same**
- you may also hear: **bringing to a common denominator**

Fractions with the same denominator are called **like fractions**.
Fractions with different denominators are called **unlike fractions**.

Here is an example without any special features:

Example

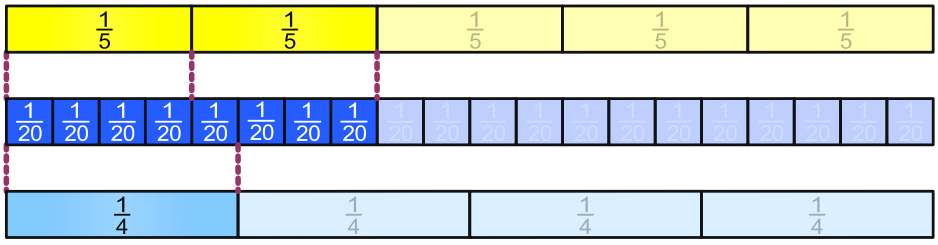
We want to write the two fractions $\frac{2}{5}$ and $\frac{1}{4}$ with the same denominator.
To do this, we look for a common multiple of the denominators 5 and 4, and we choose 20.
We multiply the numerator and denominator of $\frac{2}{5}$ by 4:

$$\frac{2}{5} = \frac{2 \times 4}{5 \times 4} = \frac{8}{20}$$

We multiply the numerator and denominator of $\frac{1}{4}$ by 5:

$$\frac{1}{4} = \frac{1 \times 5}{4 \times 5} = \frac{5}{20}$$

The two equivalent fractions now have the same denominator, 20.

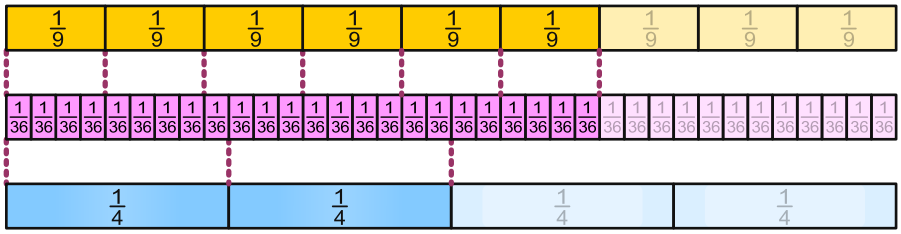


Simplify first – then rewrite with a common denominator

If we simplify the given fractions first before rewriting them with a common denominator, we usually get a smaller common denominator. This is especially practical when we want to add or subtract the rewritten fractions. Therefore, we will get used to simplifying fractions before making them like denominators.
Let us look at the fractions $\frac{6}{9}$ and $\frac{2}{4}$.
We can rewrite the fractions – without simplifying – by multiplying numerator and denominator of each fraction by the denominator of the other fraction. That is:

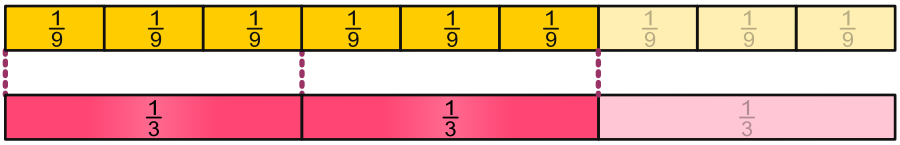
$$\frac{6}{9} = \frac{6 \times 4}{9 \times 4} = \frac{24}{36} \quad \text{and} \quad \frac{2}{4} = \frac{2 \times 9}{4 \times 9} = \frac{18}{36}\,.$$

The common denominator of the rewritten fractions is 36.

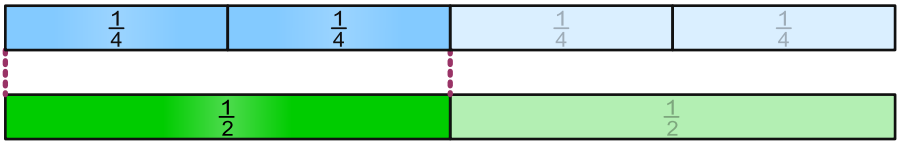


Now the fractions should be simplified first:

$$\frac{6}{9} = \frac{6 \div 3}{9 \div 3} = \frac{2}{3}$$



and $\quad \frac{2}{4} = \frac{2 \div 2}{4 \div 2} = \frac{1}{2}$.



Now we rewrite the simplified fractions with a common denominator.
A common multiple of the denominators 3 and 2 is 6.

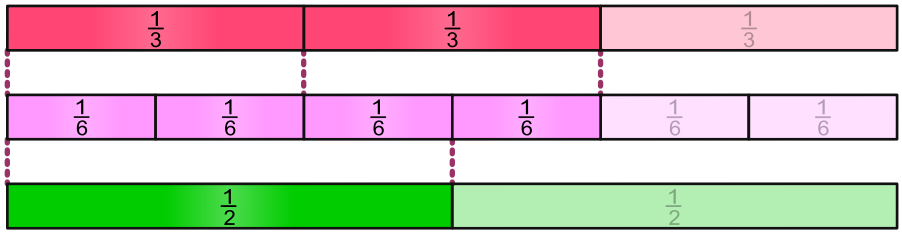
We multiply the numerator and denominator of $\frac{2}{3}$ by 2

$$\frac{2}{3} = \frac{2 \times 2}{3 \times 2} = \frac{4}{6}$$

Wemultiply the numerator and denominator of $\frac{1}{2}$ by 3

$$\frac{1}{2} = \frac{1 \times 3}{2 \times 3} = \frac{3}{6}$$

The two rewritten fractions now have the same denominator, 6.



As we can see, we get a smaller common denominator if we simplify the fractions before making them like denominators.

Rewrite with the LCM

When we rewrite two fractions with a common denominator, for example to add or subtract them, we do not want to use unnecessarily large denominators. That's why we don't rewrite the fractions with just *any* common denominator, but with the *least* common denominator. The least common denominator of two (or more) fractions is the **l**east **c**ommon **m**ultiple of all denominators (abbreviated as **LCM**).

We can see how to systematically find the least common denominator of two fractions in this example:

1. Which fractions?

We want to rewrite the fractions $\frac{3}{4}$ and $\frac{5}{6}$ with the least common denominator.

2. Simplify?

First, we check whether the fractions can be simplified and realize that this is not the case.

3. Find the LCM

Now we list the multiples of the denominators:
Multiples of 4 are: $4; 8; 12; 16; 20; 24; \ldots$
Multiples of 6 are: $6; 12; 18; 24; \ldots$
We have marked the least common multiple of the two denominators in red.

4. Divide LCM by denominator

To find out by which number we need to rewrite $\frac{3}{4}$ in order to get the least common denominator 12, we divide the least common multiple – the LCM – by the denominator of the fraction. So:

$$\frac{12}{4} = 3$$

To find out how to rewrite $\frac{5}{6}$ with the least common denominator, we also divide the LCM 12 by this denominator:

$$\frac{12}{6} = 2$$

5. Rewrite

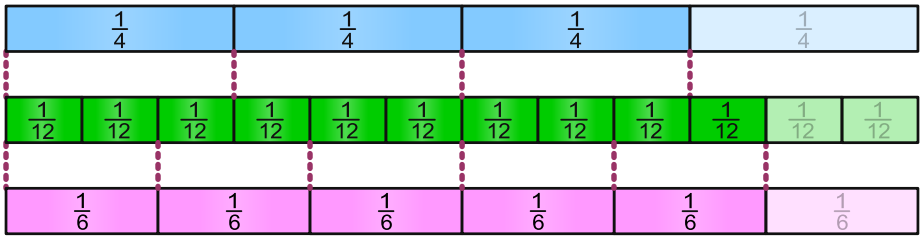
We rewrite $\frac{3}{4}$ by multiplying numerator and denominator by 3

$$\frac{3}{4} = \frac{3 \times 3}{4 \times 3} = \frac{9}{12}$$

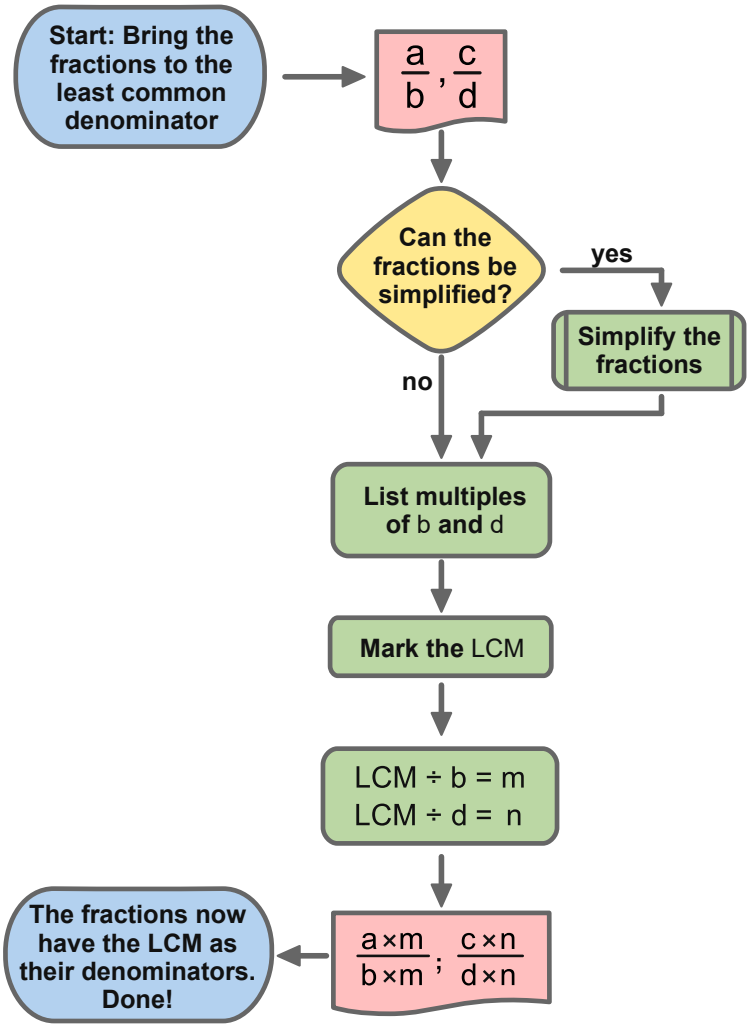
And we rewrite $\frac{5}{6}$ by multiplying numerator and denominator by 2

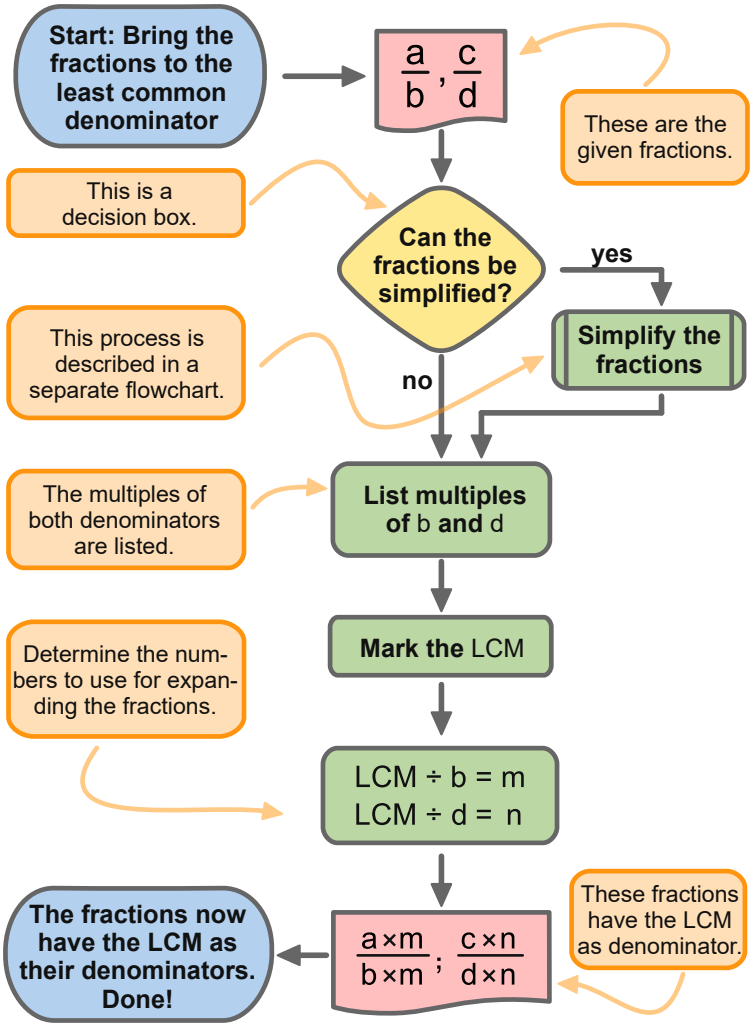
$$\frac{5}{6} = \frac{5 \times 2}{6 \times 2} = \frac{10}{12}$$

We have now rewritten both fractions with the least common denominator 12.



The process of rewriting fractions with the least common denominator can also be described with a flowchart. Below is the flowchart without comments. On the next page, the same diagram is shown with explanations.




Start: Bring the fractions to the least common denominator
$\frac{a}{b}, \frac{c}{d}$
These are the given fractions.
This is a decision box.
Can the fractions be simplified?
yes
Simplify the fractions
This process is described in a separate flowchart.
no
The multiples of both denominators are listed.
List multiples of b and d
Mark the LCM
Determine the numbers to use for expanding the fractions.
LCM ÷ b = m
LCM ÷ d = n
$\frac{a \times m}{b \times m}; \frac{c \times n}{d \times n}$
These fractions have the LCM as denominator.
The fractions now have the LCM as their denominators. Done!

Let's look at a few examples.
Example 1: without any special features

1. What fractions?

We want to rewrite the fractions $\frac{6}{7}$ and $\frac{4}{5}$ with the least common denominator.

2. Simplify?

We first check whether the fractions $\frac{6}{7}$ and $\frac{4}{5}$ can be simplified and see that they cannot be simplified further.

3. Find the least common multiple

We now list the multiples of the denominators:
Multiples of 7 are: 7, 14, 21, 28, 35, 42, 49, ...
Multiples of 5 are: 5, 10, 15, 20, 25, 30, 35, 40, 45, ...
The least common multiple of both denominators is highlighted in red.

4. Divide the least common denominator by each denominator

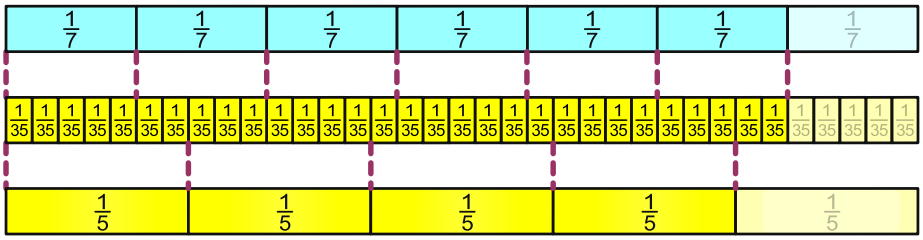
$$35 \div 7 = 5 \quad \text{and} \quad 35 \div 5 = 7$$

5. Rewrite with the least common denominator

We rewrite $\frac{6}{7}$ with denominator 35: $\frac{6}{7} = \frac{6 \times 5}{7 \times 5} = \frac{30}{35}$

And we rewrite $\frac{4}{5}$ with denominator 35: $\frac{4}{5} = \frac{4 \times 7}{5 \times 7} = \frac{28}{35}$

We have now rewritten both fractions with the least common denominator 35.



Example 2: One denominator is a multiple of the other.
We want to rewrite $\frac{2}{3}$ and $\frac{7}{9}$ with the least common denominator. Since $3 \times 3 = 9$, the denominator of $\frac{7}{9}$ is a multiple of the denominator of $\frac{2}{3}$. So 9 is the least common denominator. To rewrite both fractions with this denominator, we only need to rewrite the first fraction with denominator 9. But we can also follow our usual method step by step.

1. What fractions?

We want to rewrite the fractions $\frac{2}{3}$ and $\frac{7}{9}$ with the least common denominator.

2. Simplify?

We first check whether the fractions $\frac{2}{3}$ and $\frac{7}{9}$ can be simplified and see that they cannot be simplified further.

3. Find the least common multiple

We now list the multiples of the denominators:
Multiples of 3: 3, 6, 9, 12, 15, ...
Multiples of 9: 9, 18, 27, ...
The *least common multiple* of both denominators is highlighted in red.

4. Divide the least common denominator by each denominator

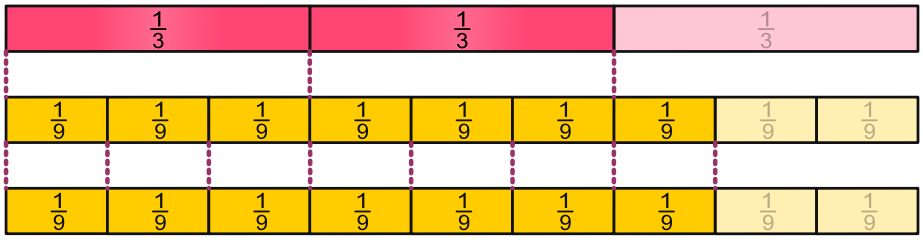
$$9 \div 3 = 3 \quad \text{and} \quad 9 \div 9 = 1$$

5. Rewrite with the least common denominator

We rewrite $\frac{2}{3}$ with denominator 9: $\frac{2}{3} = \frac{2 \times 3}{3 \times 3} = \frac{6}{9}$

And we rewrite $\frac{7}{9}$ with denominator 9: $\frac{7}{9} = \frac{7 \times 1}{9 \times 1} = \frac{7}{9}$

We have now rewritten both fractions with the least common denominator 9.



Example 3: The denominators have common factors.
We want to rewrite $\frac{3}{8}$ and $\frac{5}{6}$ with the least common denominator. Since both denominators have common factors, the least common denominator is smaller than the product of the two denominators.

1. What fractions?

We want to rewrite the fractions $\frac{3}{8}$ and $\frac{5}{6}$ with the least common denominator.

2. Simplify?

We first check whether the fractions $\frac{3}{8}$ and $\frac{5}{6}$ can be simplified and find that they cannot.

3. Find the least common multiple

We now list the multiples of the denominators:
Multiples of 8: 8, 16, 24, 32, 40, ...
Multiples of 6: 6, 12, 18, 24, 30, 36, ...
The *least common multiple* of both denominators is highlighted in red.

4. Divide the least common denominator by each denominator

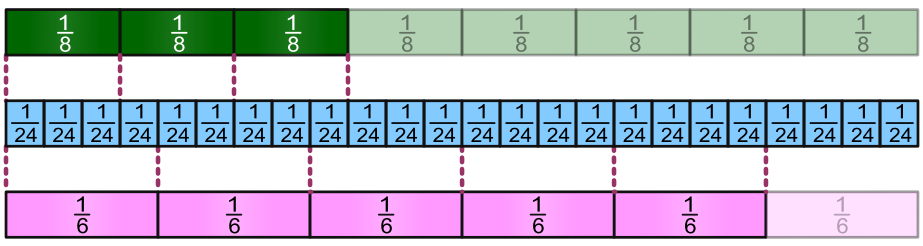
$24 \div 8 = 3$ and $24 \div 6 = 4$

5. Rewrite with the least common denominator

We rewrite $\frac{3}{8}$ with denominator 24: $\frac{3}{8} = \frac{3 \times 3}{8 \times 3} = \frac{9}{24}$

And we rewrite $\frac{5}{6}$ with denominator 24: $\frac{5}{6} = \frac{5 \times 4}{6 \times 4} = \frac{20}{24}$

We have now rewritten both fractions with the least common denominator 24.



Example 4: One of the fractions can be simplified.
We want to rewrite $\frac{10}{15}$ and $\frac{7}{10}$ with the least common denominator. In this case, simplifying the first fraction does not change the least common denominator, because the LCM of 15 and 10 is 30, which is the same as the LCM of 3 and 10.

1. What fractions?

We want to rewrite the fractions $\frac{10}{15}$ and $\frac{7}{10}$ with the least common denominator.

2. Simplify?

We simplify the first fraction: $\frac{10}{15} = \frac{10\div 5}{15\div 5} = \frac{2}{3}$

From now on, we look for the least common denominator of $\frac{2}{3}$ and $\frac{7}{10}$.

3. Find the least common multiple

Now we list the multiples of the denominators:
Multiples of 3: 3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, ...
Multiples of 10: 10, 20, 30, 40, ...
The *least common multiple* of both denominators is highlighted in red.

4. Divide the least common denominator by each denominator

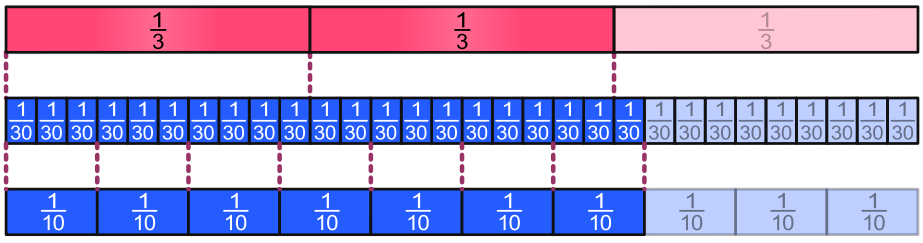
$30 \div 3 = 10$ and $30 \div 10 = 3$

5. Rewrite with the least common denominator

We rewrite $\frac{2}{3}$ with denominator 30: $\frac{2}{3} = \frac{2\times 10}{3\times 10} = \frac{20}{30}$

And we rewrite $\frac{7}{10}$ with denominator 30: $\frac{7}{10} = \frac{7\times 3}{10\times 3} = \frac{21}{30}$

We have now rewritten both fractions with the least common denominator 30.



Example 5: Both fractions can be simplified.
We want to rewrite $\frac{7}{35}$ and $\frac{55}{66}$ with the least common denominator. If we didn't simplify and just multiplied the denominators, the common denominator would be 2310, which would be rather inconvenient for further calculations.

1. What fractions?

We want to rewrite the fractions $\frac{7}{35}$ and $\frac{55}{66}$ with the least common denominator.

2. Simplify?

We simplify the fractions: $\frac{7}{35} = \frac{7\div 7}{35\div 7} = \frac{1}{5}$; $\frac{55}{66} = \frac{55\div 11}{66\div 11} = \frac{5}{6}$

Now we look for the least common denominator of $\frac{1}{5}$ and $\frac{5}{6}$.

3. Find the least common multiple

Now we list the multiples of the denominators:
Multiples of 5: 5, 10, 15, 20, 25, 30, 35, ...
Multiples of 6: 6, 12, 18, 24, 30, 36, ...
The *least common multiple* of both denominators is highlighted in red.

4. Divide the least common denominator by each denominator

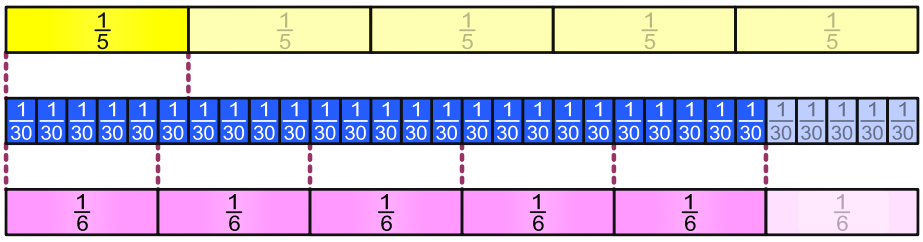
$30 \div 5 = 6$ and $30 \div 6 = 5$

5. Rewrite with the least common denominator

We rewrite $\frac{1}{5}$ with denominator 30: $\frac{1}{5} = \frac{1\times 6}{5\times 6} = \frac{6}{30}$

And we rewrite $\frac{5}{6}$ with denominator 30: $\frac{5}{6} = \frac{5\times 5}{6\times 5} = \frac{25}{30}$

We have now rewritten both fractions with the least common denominator 30.



Example 6: The denominators of the simplified fractions share common factors. We want to rewrite $\frac{28}{40}$ and $\frac{39}{75}$ with the least common denominator. The denominators of the simplified fractions share common factors, so the least common multiple is smaller than the product of the two denominators.

1. What fractions?

We want to rewrite the fractions $\frac{28}{40}$ and $\frac{39}{75}$ with the least common denominator.

2. Simplify?

We simplify the fractions: $\frac{28}{40} = \frac{28\div 4}{40\div 4} = \frac{7}{10}$; $\frac{39}{75} = \frac{39\div 3}{75\div 3} = \frac{13}{25}$

Now we look for the least common denominator of $\frac{7}{10}$ and $\frac{13}{25}$.

3. Find the least common multiple

Now we list the multiples of the denominators:
Multiples of 10: 10, 20, 30, 40, 50, 60, ...
Multiples of 25: 25, 50, 75, ...
The *least common multiple* of both denominators is highlighted in red.

4. Divide the least common denominator by each denominator

$50 \div 10 = 5$ and $50 \div 25 = 2$

5. Rewrite with the least common denominator

We rewrite $\frac{7}{10}$ with denominator 50: $\frac{7}{10} = \frac{7\times 5}{10\times 5} = \frac{35}{50}$

And we rewrite $\frac{13}{25}$ with denominator 50: $\frac{13}{25} = \frac{13\times 2}{25\times 2} = \frac{26}{50}$

We have now rewritten both fractions with the least common denominator 50.