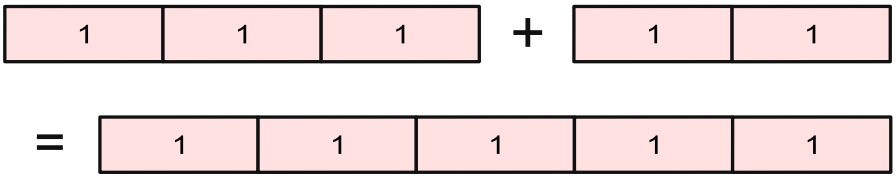
Adding Fractions

Adding Like Fractions

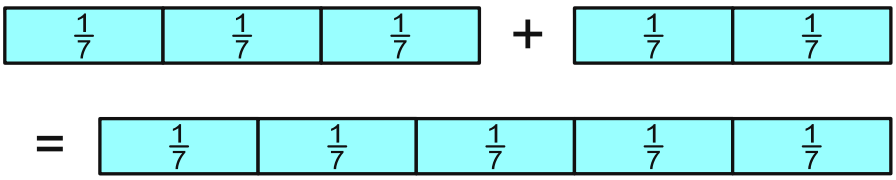
We know how to add natural numbers. For example, we calculate: $3+2=5$.
We can imagine this using number strips:



We want to add fractions in a similar way. For example, we want the following to be valid:

$$\frac{3}{7}+\frac{2}{7}=\frac{5}{7}$$

.
Here is how this looks using fraction strips:



As we can see, we can add like fractions by adding the numerators and keeping the denominators. Therefore, we define:

> Like fractions are added
> by adding the numerators.
> The denominators remain the same.

We can write this relationship also as a formula:

$$\frac{\mathbf{a}}{\mathbf{c}}+\frac{\mathbf{b}}{\mathbf{c}}=\frac{\mathbf{a}+\mathbf{b}}{\mathbf{c}}$$

Whenever we replace the variables with numbers, we obtain a true equation.
For example:
If we replace **a** with **2**, **b** with **5**, and **c** with **3**, we obtain the true equation:

$$\frac{\mathbf{2}}{\mathbf{3}}+\frac{\mathbf{5}}{\mathbf{3}}=\frac{\mathbf{2}+\mathbf{5}}{\mathbf{3}}=\frac{7}{3}$$

We can also replace **a** with **7**, **b** with **1**, and **c** with **9**. Again, we obtain a true equation:

$$\frac{7}{9}+\frac{1}{9}=\frac{7+1}{9}=\frac{8}{9}$$

Adding unlike fractions

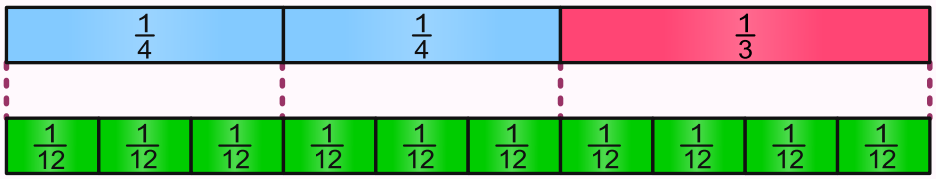
Problem: When we want to add unlike fractions, we have to take a different approach, since the denominators cannot remain the same — they are different. For example:

Solution: We convert the fractions to a common denominator and then add the equivalent fractions. Here is the reasoning:

The result of, for example, $\frac{2}{4}+\frac{1}{3}$ should represent the total length of the two fraction strips.

We arrive at the same total length if we replace $\frac{2}{4}$ and $\frac{1}{3}$ with other fractions of the same size.

For example, we can replace $\frac{2}{4}$ with $\frac{6}{12}$ and $\frac{1}{3}$ with $\frac{4}{12}$.



Since $\frac{6}{12}$ and $\frac{4}{12}$ have the same denominator, we can add the two fractions by adding the numerators and keeping the denominators the same.

So: $\frac{6}{12}+\frac{4}{12}=\frac{10}{12}$.

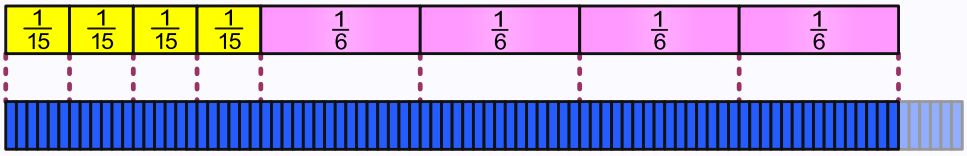
When adding fractions, there are three more important points to consider:

1. Least Common Denominator

If we want to add two unlike fractions, we could find a common denominator by multiplying the denominators of the two fractions. However, this may lead to unnecessarily large numbers. Therefore, we usually rewrite fractions with the least common multiple of the denominators — that is, the least common denominator. Let's look at an example:

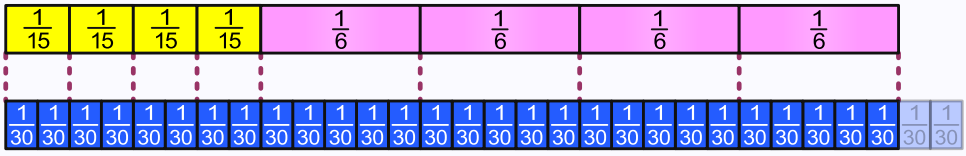
If we want to add the fractions $\frac{4}{15}$ and $\frac{4}{6}$, we could multiply each fraction by the denominator of the other fraction. Then we would have to work with ninetyths:

$$\frac{4}{15}+\frac{4}{6}=\frac{4\times 6}{15\times 6}+\frac{4\times 15}{6\times 15}=\frac{24}{90}+\frac{60}{90}=\frac{84}{90}$$



However, since the least common multiple (LCM) of 15 and 6 is 30, it is sufficient to rewrite both fractions using thirtieths:

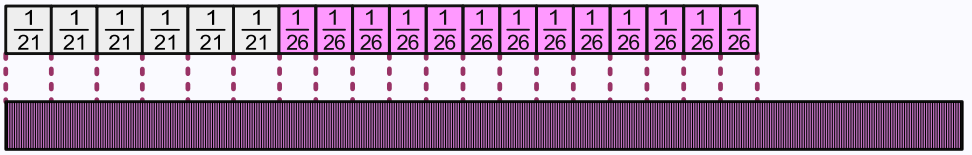
$$\frac{4}{15}+\frac{4}{6}=\frac{4\times 2}{15\times 2}+\frac{4\times 5}{6\times 5}=\frac{8}{30}+\frac{20}{30}=\frac{28}{30}$$



2. At the Beginning: Simplify First

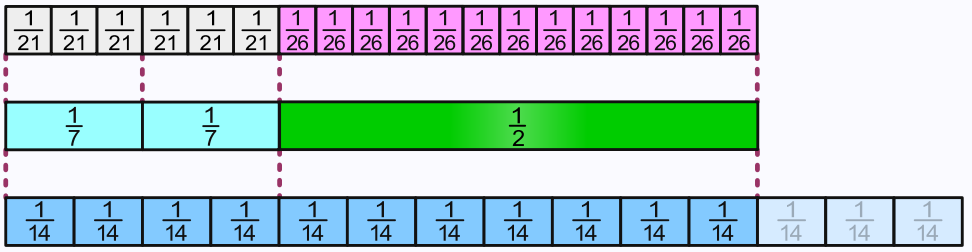
To avoid working with unnecessarily large numbers, we first simplify both fractions as much as possible before adding them.
Let's look at the benefit with an example:
If we want to add the fractions $\frac{6}{21}$ and $\frac{13}{26}$, we could rewrite both fractions using the LCM of 21 and 26. Then we would have to work with five-hundred-forty-sixths:

$$\frac{6}{21}+\frac{13}{26}=\frac{6\times 26}{21\times 26}+\frac{13\times 21}{26\times 21}=\frac{156}{546}+\frac{273}{546}=\frac{429}{546}$$



However, if we simplify first, we end up with a much smaller least common denominator.

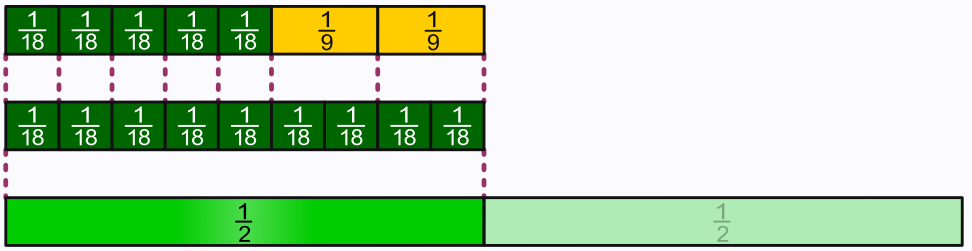
$$\frac{6}{21}+\frac{13}{26}=\frac{6\div 3}{21\div 3}+\frac{13\div 13}{26\div 13}=\frac{2}{7}+\frac{1}{2}=\frac{2\times 2}{7\times 2}+\frac{1\times 7}{2\times 7}=\frac{4}{14}+\frac{7}{14}=\frac{11}{14}$$



3. At the End: Simplify

After we add fractions, we write the result in its simplest form. That means: If we can simplify the result, we do it. For example:

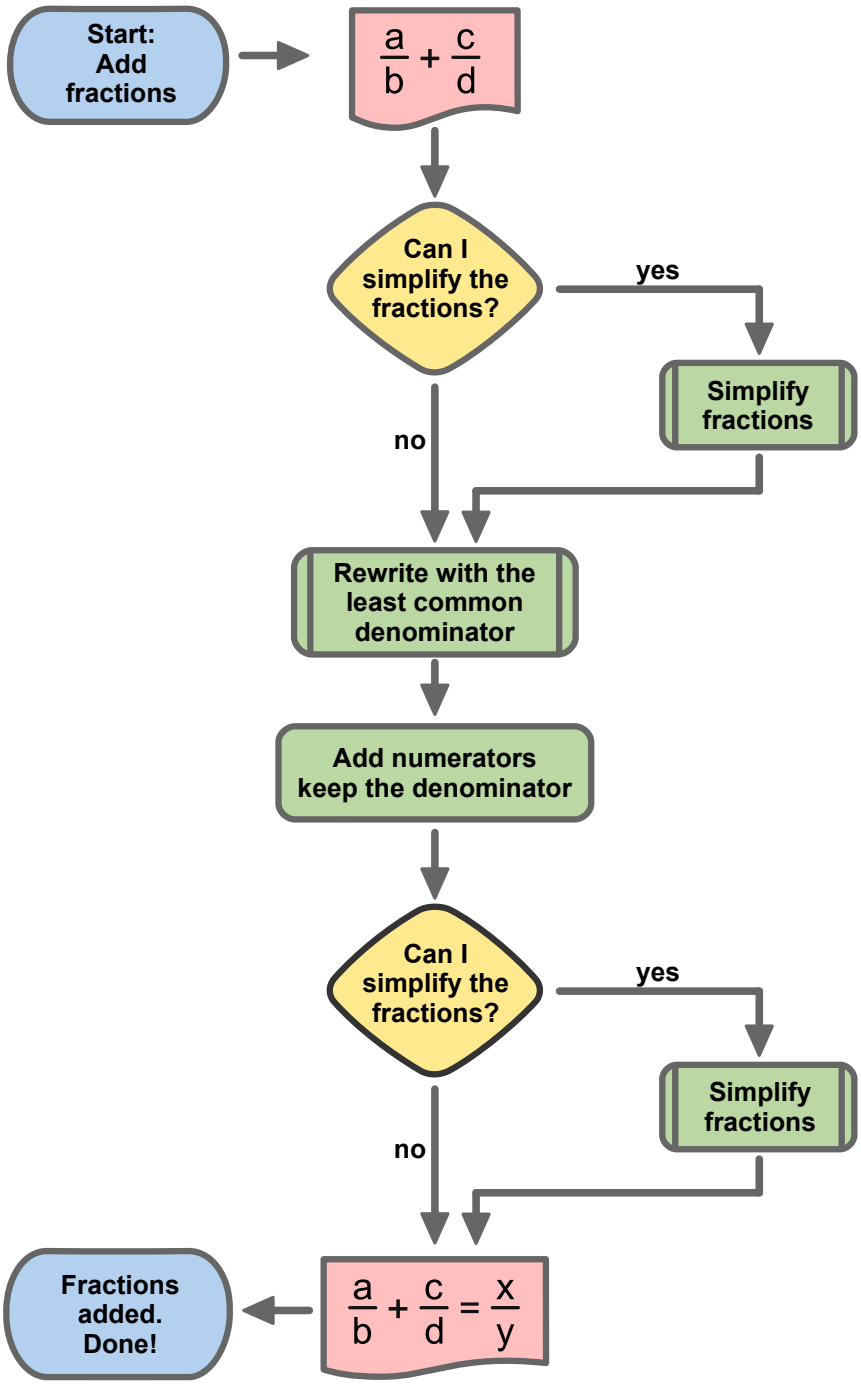
$$\frac{5}{18}+\frac{2}{9}=\frac{5}{18}+\frac{2\times 2}{9\times 2}=\frac{5}{18}+\frac{4}{18}=\frac{9}{18}=\frac{9\div 9}{18\div 9}=\frac{1}{2}$$



If the result is a whole number, we write that number instead of a fraction. For example:

$$\frac{3}{4}+\frac{5}{4}=\frac{3+5}{4}=\frac{8}{4}=\frac{8\div 4}{4\div 4}=\frac{2}{1}=2$$

The process of adding fractions with all necessary steps is shown in the following flow-chart.



Let's look at some examples.

1. Goal?

We want to add the two fractions $\frac{1}{8}$ and $\frac{3}{9}$.

2. Simplify?

$\frac{3}{9} = \frac{3\div 3}{9\div 3} = \frac{1}{3}$ From now on, we will add $\frac{1}{8}$ and $\frac{1}{3}$.

3. Least Common Denominator

$$\frac{1}{8} = \frac{1\times 3}{8\times 3} = \frac{3}{24} \quad , \quad \frac{1}{3} = \frac{1\times 8}{3\times 8} = \frac{8}{24}$$

4. Add

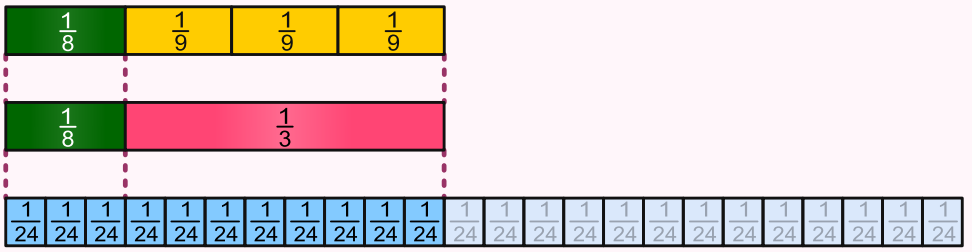
$$\frac{3}{24} + \frac{8}{24} = \frac{11}{24}$$

5. Simplify?

$\frac{11}{24}$ cannot be simplified.

6. Final Result

$$\frac{1}{8} + \frac{3}{9} = \frac{1}{8} + \frac{3\div 3}{9\div 3} = \frac{1}{8} + \frac{1}{3} = \frac{1\times 3}{8\times 3} + \frac{1\times 8}{3\times 8} = \frac{3}{24} + \frac{8}{24} = \frac{11}{24}$$



1. Goal?

We want to add the two fractions $\frac{7}{9}$ and $\frac{1}{2}$.

2. Simplify?

$\frac{7}{9}$ and $\frac{1}{2}$ cannot be simplified.

3. Least Common Denominator

$$\frac{7}{9} = \frac{7\times 2}{9\times 2} = \frac{14}{18} \quad , \quad \frac{1}{2} = \frac{1\times 9}{2\times 9} = \frac{9}{18}$$

4. Add

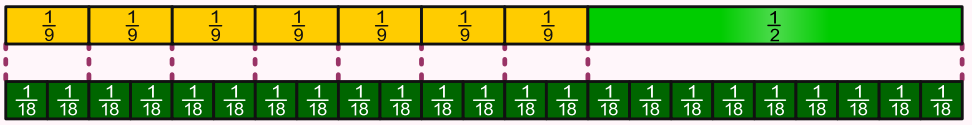
$$\frac{14}{18} + \frac{9}{18} = \frac{23}{18}$$

5. Simplify?

$\frac{23}{18}$ cannot be simplified.

6. Final Result

$$\frac{7}{9} + \frac{1}{2} = \frac{7\times 2}{9\times 2} + \frac{1\times 9}{2\times 9} = \frac{14}{18} + \frac{9}{18} = \frac{23}{18}$$



1. Goal?

We want to add the two fractions $\frac{3}{18}$ and $\frac{6}{20}$.

2. Simplify?

$$\frac{3}{18} = \frac{3\div 3}{18\div 3} = \frac{1}{6} \quad \text{and} \quad \frac{6}{20} = \frac{6\div 2}{20\div 2} = \frac{3}{10}$$

3. Least Common Denominator

$$\frac{1}{6} = \frac{1\times 5}{6\times 5} = \frac{5}{30} \quad , \quad \frac{3}{10} = \frac{3\times 3}{10\times 3} = \frac{9}{30}$$

4. Add

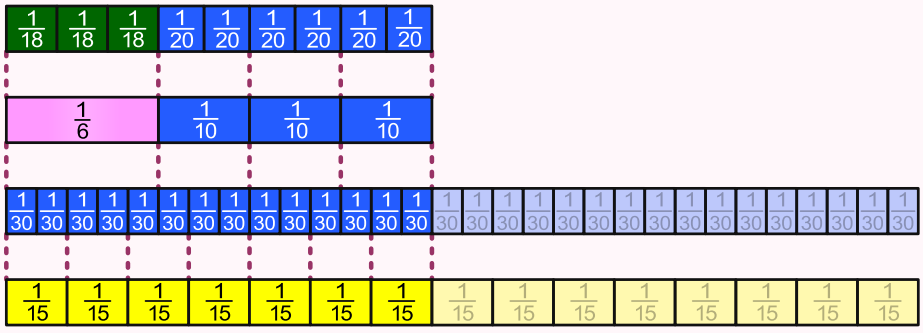
$$\frac{5}{30} + \frac{9}{30} = \frac{14}{30}$$

5. Simplify?

$$\frac{14}{30} = \frac{14\div 2}{30\div 2} = \frac{7}{15}$$

6. Final Result

$$\frac{3}{18} + \frac{6}{20} = \frac{3\div 3}{18\div 3} + \frac{6\div 2}{20\div 2} = \frac{1}{6} + \frac{3}{10} = \frac{1\times 5}{6\times 5} + \frac{3\times 3}{10\times 3} = \frac{5}{30} + \frac{9}{30} = \frac{14}{30} = \frac{14\div 2}{30\div 2} = \frac{7}{15}$$



Exercise 1

1. Goal? We want to add the fractions $\frac{21}{56}$ and $\frac{10}{16}$.

2. Simplify? $\frac{21}{56} = \frac{21\div 7}{56\div 7} = \frac{3}{8}$ and $\frac{10}{16} = \frac{10\div 2}{16\div 2} = \frac{5}{8}$

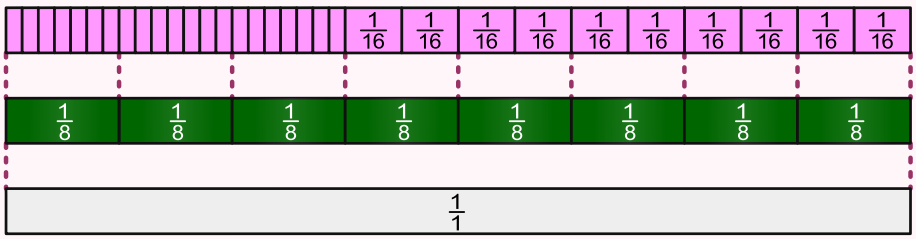
3. Common Denominator $\frac{3}{8}$, $\frac{5}{8}$

4. Add $\frac{3}{8} + \frac{5}{8} = \frac{8}{8}$

5. Simplify? $\frac{8}{8} = \frac{8\div 8}{8\div 8} = \frac{1}{1} = 1$

6. Result

$$\frac{21}{56} + \frac{10}{16} = \frac{21\div 7}{56\div 7} + \frac{10\div 2}{16\div 2} = \frac{3}{8} + \frac{5}{8} = \frac{8}{8} = \frac{8\div 8}{8\div 8} = \frac{1}{1} = 1$$



Exercise 2

1. Goal? We want to add the fractions $\frac{6}{21}$ and $\frac{6}{28}$.

2. Simplify? $\frac{6}{21} = \frac{6\div 3}{21\div 3} = \frac{2}{7}$ and $\frac{6}{28} = \frac{6\div 2}{28\div 2} = \frac{3}{14}$

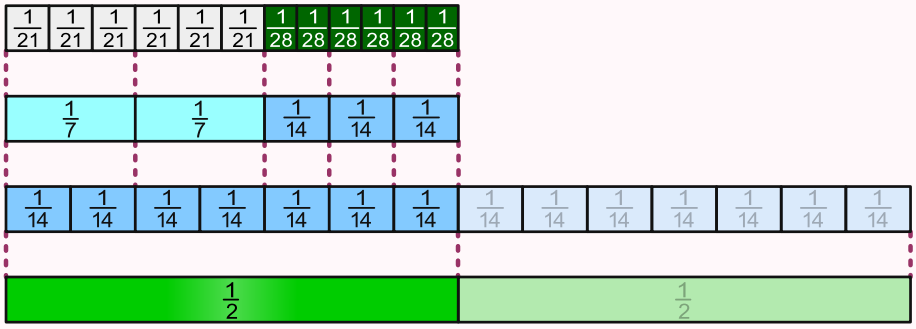
3. Common Denominator $\frac{2}{7} = \frac{2\times 2}{7\times 2} = \frac{4}{14}$, $\frac{3}{14}$

4. Add $\frac{4}{14} + \frac{3}{14} = \frac{7}{14}$

5. Simplify? $\frac{7}{14} = \frac{7\div 7}{14\div 7} = \frac{1}{2}$

6. Result

$$\frac{6}{21}+\frac{6}{28}=\frac{6\div 3}{21\div 3}+\frac{6\div 2}{28\div 2}=\frac{2}{7}+\frac{3}{14}=\frac{2\times 2}{7\times 2}+\frac{3}{14}=\frac{4}{14}+\frac{3}{14}=\frac{7}{14}=\frac{7\div 7}{14\div 7}=\frac{1}{2}$$



Exercise 3

1. Goal? We want to add the fractions $\frac{12}{45}$ and $\frac{10}{12}$.

2. Simplify? $\frac{12}{45} = \frac{12\div 3}{45\div 3} = \frac{4}{15}$ and $\frac{10}{12} = \frac{10\div 2}{12\div 2} = \frac{5}{6}$

3. Common Denominator

$$\frac{4}{15} = \frac{4\times 2}{15\times 2} = \frac{8}{30} \quad , \quad \frac{5}{6} = \frac{5\times 5}{6\times 5} = \frac{25}{30}$$

4. Add

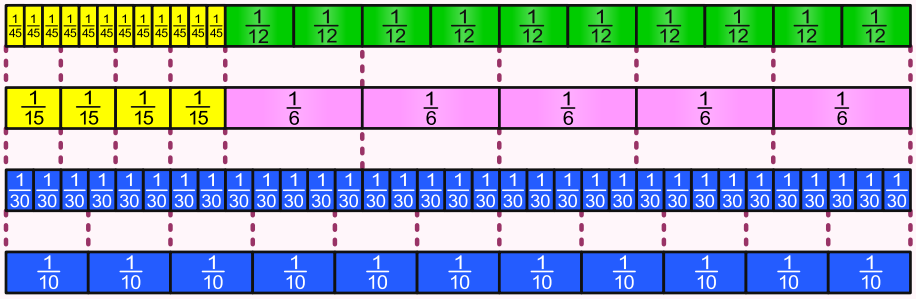
$$\frac{8}{30} + \frac{25}{30} = \frac{33}{30}$$

5. Simplify?

$$\frac{33}{30} = \frac{33\div 3}{30\div 3} = \frac{11}{10}$$

6. Result

$$\frac{12}{45} + \frac{10}{12} = \frac{12\div 3}{45\div 3} + \frac{10\div 2}{12\div 2} = \frac{4}{15} + \frac{5}{6} = \frac{4\times 2}{15\times 2} + \frac{5\times 5}{6\times 5} = \frac{8}{30} + \frac{25}{30} = \frac{33}{30} = \frac{33\div 3}{30\div 3} = \frac{11}{10}$$



Exercise 4

1. Goal? We want to add the fractions $\frac{6}{27}$ and $\frac{1}{36}$.

2. Simplify? $\frac{6}{27} = \frac{6\div 3}{27\div 3} = \frac{2}{9}$ and $\frac{1}{36}$

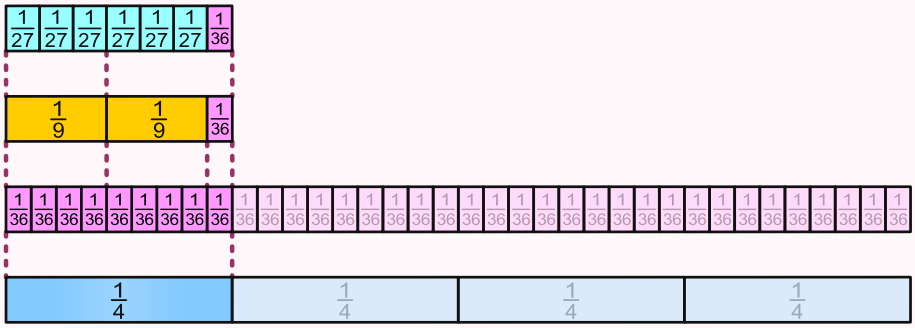
3. Common Denominator $\frac{2}{9} = \frac{2\times 4}{9\times 4} = \frac{8}{36}$, $\frac{1}{36}$

4. Add $\frac{8}{36} + \frac{1}{36} = \frac{9}{36}$

5. Simplify? $\frac{9}{36} = \frac{9\div 9}{36\div 9} = \frac{1}{4}$

6. Result

$$\frac{6}{27} + \frac{1}{36} = \frac{6\div 3}{27\div 3} + \frac{1}{36} = \frac{2}{9} + \frac{1}{36} = \frac{2\times 4}{9\times 4} + \frac{1}{36} = \frac{8}{36} + \frac{1}{36} = \frac{9}{36} = \frac{9\div 9}{36\div 9} = \frac{1}{4}$$



Exercise 5

1. Goal? We want to add the fractions $\frac{6}{105}$ and $\frac{27}{42}$.

2. Simplify? $\frac{6}{105} = \frac{6\div 3}{105\div 3} = \frac{2}{35}$ and $\frac{27}{42} = \frac{27\div 3}{42\div 3} = \frac{9}{14}$

3. Common Denominator

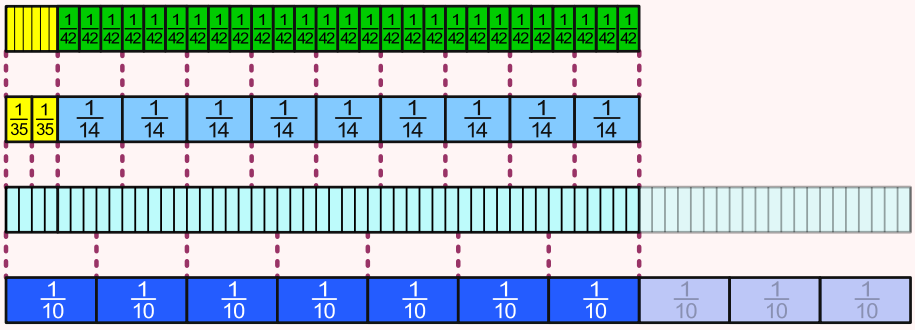
$$\frac{2}{35} = \frac{2\times 2}{35\times 2} = \frac{4}{70} \quad , \quad \frac{9}{14} = \frac{9\times 5}{14\times 5} = \frac{45}{70}$$

4. Add $\frac{4}{70} + \frac{45}{70} = \frac{49}{70}$

5. Simplify? $\frac{49}{70} = \frac{49\div 7}{70\div 7} = \frac{7}{10}$

6. Result

$$\frac{6}{105}+\frac{27}{42}=\frac{6\div 3}{105\div 3}+\frac{27\div 3}{42\div 3}=\frac{2}{35}+\frac{9}{14}=\frac{2\times 2}{35\times 2}+\frac{9\times 5}{14\times 5}=\frac{4}{70}+\frac{45}{70}=\frac{49}{70}=\frac{49\div 7}{70\div 7}=\frac{7}{10}$$



Exercise 6

1. Goal? We want to add the fractions $\frac{8}{42}$ and $\frac{26}{39}$.

2. Simplify? $\frac{8}{42} = \frac{8\div 2}{42\div 2} = \frac{4}{21}$ and $\frac{26}{39} = \frac{26\div 13}{39\div 13} = \frac{2}{3}$

3. Common denominator $\frac{4}{21}$, $\frac{2}{3} = \frac{2\times 7}{3\times 7} = \frac{14}{21}$

4. Add $\frac{4}{21} + \frac{14}{21} = \frac{18}{21}$

5. Simplify? $\frac{18}{21} = \frac{18\div 3}{21\div 3} = \frac{6}{7}$

6. Final result

$$\frac{8}{42} + \frac{26}{39} = \frac{8\div 2}{42\div 2} + \frac{26\div 13}{39\div 13} = \frac{4}{21} + \frac{2}{3} = \frac{4}{21} + \frac{2\times 7}{3\times 7} = \frac{4}{21} + \frac{14}{21} = \frac{18}{21} = \frac{18\div 3}{21\div 3} = \frac{6}{7}$$