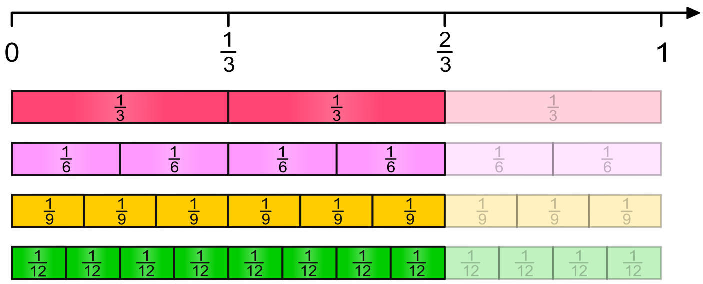
Comparing Fractions

Equivalent Fractions

For every fraction we find on the number line, there are other fractions that are equally large. For example: $\mathbf{\frac{2}{3}} = \mathbf{\frac{4}{6}} = \mathbf{\frac{8}{12}} = \mathbf{\frac{10}{15}} = \mathbf{\frac{12}{18}} =$ and so on.
If we write an equivalent fraction for $\mathbf{\frac{2}{3}}$ by multiplying numerator and denominator by 7, 8, 9, ..., we obtain other fractions of the same value.

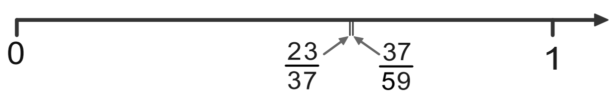


We can imagine it like this on the number line.

At every point where a natural number is located, there are also many fractions. $1 = \frac{1}{1} = \frac{2}{2} = \frac{3}{3} = \frac{4}{4} = \frac{5}{5} = \frac{6}{6} =$ and so on.

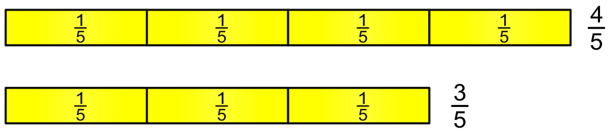
Comparing Fractions

We can always tell which of two natural numbers is greater. For example, we immediately know that 89 is greater than 78 without having to think about it.
But when we are given the fractions $\frac{8}{9}$ and $\frac{7}{8}$, it is not so obvious.
If we want to know whether two fractions are equal or which one is greater, we could plot the fractions on the number line. If both fractions are located at the same point, they are equal. If one of the fractions is to the right of the other, then the one on the right is greater and the one on the left is smaller.
But this method can be very tedious. If the fractions are close together, the difference may be hard to see, as with $\frac{23}{37}$ and $\frac{37}{59}$.

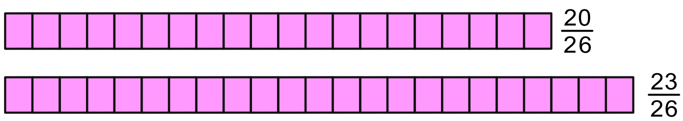


Therefore, we need a method to quickly and reliably compare two fractions. One possible solution is to bring the fractions to the same denominator and then compare the numerators.

Explanation: When two fractions have the same denominator, we call them **like fractions**. Like fractions can be compared by comparing their numerators. For example: $\frac{4}{5}$ is greater than $\frac{3}{5}$ — written as $\frac{4}{5} > \frac{3}{5}$ (read as four fifths greater than three fifths") — because 4 is greater than 3.



Even if the denominators are quite large, we can still compare the fractions easily. $\frac{20}{26} < \frac{23}{26}$ (read as: “twenty twenty-sixths is less than twenty-three twenty-sixths”).



If we want to compare two like fractions, the size of the denominators does not matter at all. As long as the *numerator* of one fraction is greater than that of the other, the *entire fraction* is greater than the other.

$$\frac{20}{\text{any denominator}} < \frac{23}{\text{any denominator}}$$

Let’s summarize our method:

How to compare fractions:
1) Bring the fractions to a common denominator.
2) Compare the numerators: The fraction with the largest numerator is the greatest, the fraction with the smallest numerator is the smallest.

Example 1

We want to compare the fractions $\frac{13}{31}$ and $\frac{17}{31}$.

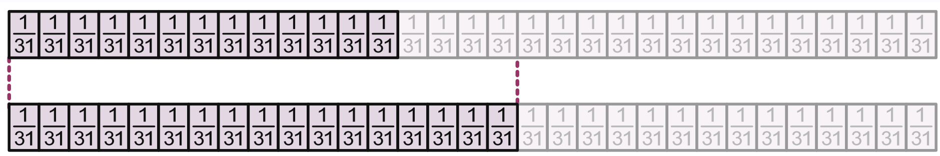
Since both fractions have the same denominator, we do not need to bring the fractions to a common denominator.

We have $\frac{13}{31}$ less than $\frac{17}{31}$.

We can write this more concisely as:

$$\frac{13}{31} < \frac{17}{31}$$

Read as: Thirteen thirty-firsts is less than seventeen thirty-firsts.



Example 2

We want to compare the fractions $\frac{5}{7}$ and $\frac{2}{3}$.

The least common denominator is the **LCM** of 7 and 3, which is 21. We convert:

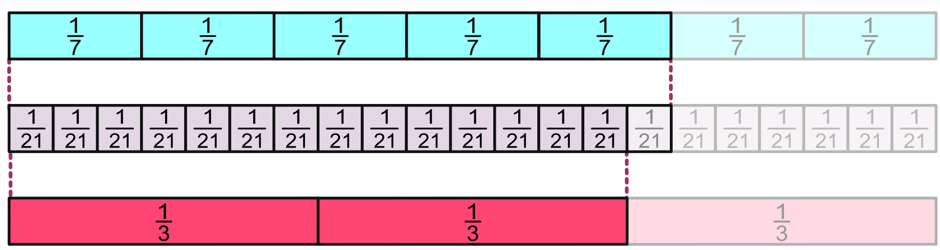
$$\frac{5}{7} = \frac{5 \times 3}{7 \times 3} = \frac{15}{21} \quad ; \quad \frac{2}{3} = \frac{2 \times 7}{3 \times 7} = \frac{14}{21}$$

Since $\frac{15}{21}$ is greater than $\frac{14}{21}$, it follows that $\frac{5}{7}$ is greater than $\frac{2}{3}$.

We can write this more concisely as:

$$\frac{15}{21} = \frac{5}{7} > \frac{2}{3} = \frac{14}{21}$$

Read as: Fifteen twenty-firsts equals five sevenths is greater than two thirds equals fourteen twenty-firsts.



Example 3

We want to compare the fractions $\frac{19}{24}$ and $\frac{13}{16}$.

The least common denominator is the **LCM** of 24 and 16, which is 48.

We convert:

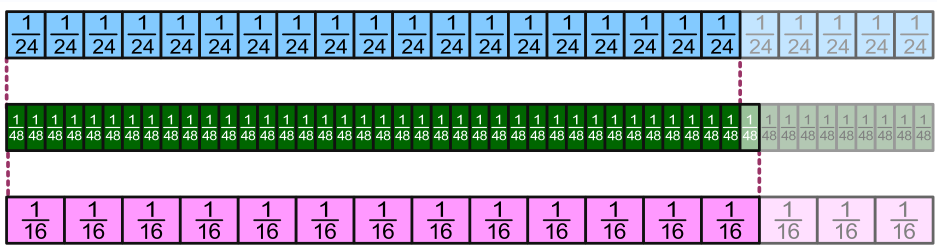
$$\frac{19}{24} = \frac{19 \times 2}{24 \times 2} = \frac{38}{48} \quad ; \quad \frac{13}{16} = \frac{13 \times 3}{16 \times 3} = \frac{39}{48}$$

Because $\frac{38}{48}$ is less than $\frac{39}{48}$, $\frac{19}{24}$ is less than $\frac{13}{16}$.

We can write this more concisely as:

$$\frac{38}{48} = \frac{19}{24} < \frac{13}{16} = \frac{39}{48}$$

Read as: Thirty-eight forty-eighths equals nineteen twenty-fourths is less than thirteen sixteenths equals thirty-nine forty-eighths.



Example 4

We want to compare the fractions $\frac{5}{8}$ and $\frac{5}{9}$.

The least common denominator is the **LCM** of 8 and 9, which is 72. We convert:

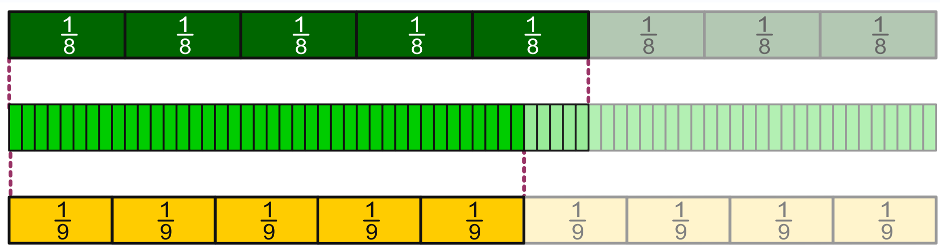
$$\frac{5}{8} = \frac{5 \times 9}{8 \times 9} = \frac{45}{72} \quad ; \quad \frac{5}{9} = \frac{5 \times 8}{9 \times 8} = \frac{40}{72}$$

Because $\frac{45}{72}$ is greater than $\frac{40}{72}$, $\frac{5}{8}$ is greater than $\frac{5}{9}$.

We can write this more concisely as:

$$\frac{45}{72} = \frac{5}{8} > \frac{5}{9} = \frac{40}{72}$$

Read as: Forty-five seventy-seconds equals five eighths is greater than five ninths equals forty seventy-seconds.



We could also consider the following in this case:

The smaller the denominator of a fraction, the larger the parts.

Thus, $\frac{1}{8}$ is greater than $\frac{1}{9}$.

Therefore, $\frac{5}{8}$ is also greater than $\frac{5}{9}$.

In general: If two fractions have the same numerator, the fraction with the smaller denominator is the larger one.

Example 5

We want to compare the fractions $\frac{6}{7}$ and $\frac{5}{6}$.

The least common denominator is the **LCM** of 7 and 6, which is 42. We convert:

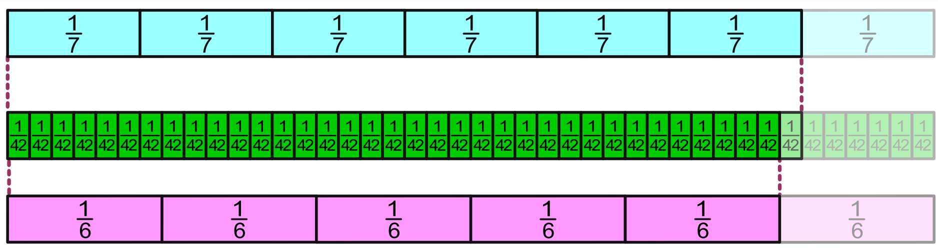
$$\frac{6}{7} = \frac{6 \cdot 6}{7 \cdot 6} = \frac{36}{42} \quad ; \quad \frac{5}{6} = \frac{5 \cdot 7}{6 \cdot 7} = \frac{35}{42}$$

Since $\frac{36}{42}$ is greater than $\frac{35}{42}$, $\frac{6}{7}$ is greater than $\frac{5}{6}$.

We can write this more concisely as:

$$\frac{36}{42} = \frac{6}{7} > \frac{5}{6} = \frac{35}{42}$$

Read as: Thirty-six forty-seconds equals six sevenths is greater than five sixths equals thirty-five forty-seconds.



In this case, we could also have reasoned as follows:

To reach one whole from $\frac{6}{7}$ we are missing $\frac{1}{7}$.

To reach one whole from $\frac{5}{6}$ we are missing $\frac{1}{6}$.

Since $\frac{1}{6}$ is greater than $\frac{1}{7}$, more is missing from $\frac{5}{6}$ than from $\frac{6}{7}$ to make one whole.

Therefore, $\frac{6}{7}$ is greater than $\frac{5}{6}$.