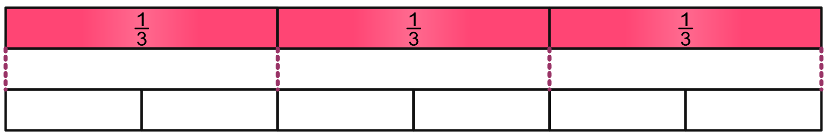
Equivalent Fractions

We can often make calculations with fractions easier by not working with the given fractions directly, but instead using other fractions that are equivalent in size.
That's why there are several procedures for finding a different fraction that is the same size as a given one. Here is one:

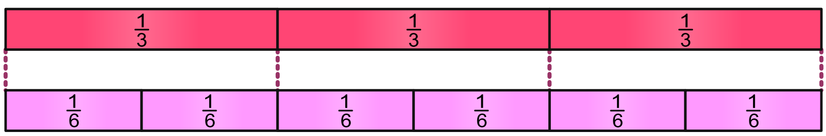
Creating Equivalent Fractions. Arithmetically, we create an equivalent fraction by multiplying both the numerator and the denominator by the same natural number. Visually, we create an equivalent fraction by dividing the parts of the original fraction into smaller equal parts. Let's look at some examples.

Example 1

If we divide each third into two equal parts, there are $3 \times 2 = 6$ equal parts on this fraction strip.

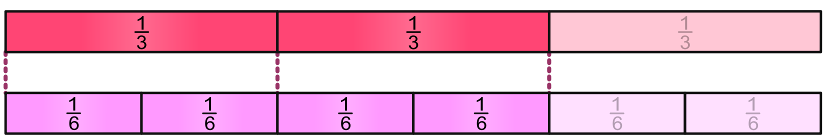


These are sixths.



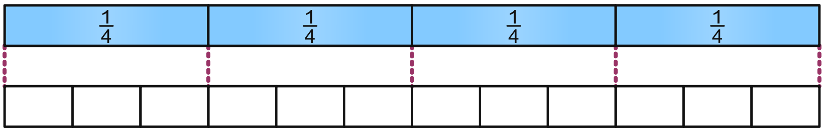
Since we divided each third into 2 equal parts, each of these parts is only half as large as $\frac{1}{3}$.
So for a fraction with denominator 6 that is supposed to be the same size as a fraction with denominator 3, we need twice as many parts.
Therefore, if we multiply the numerator and the denominator of $\frac{2}{3}$ by 2, we get an equivalent fraction, namely $\frac{4}{6}$.
We have:

$$\frac{2}{3} = \frac{2 \times 2}{3 \times 2} = \frac{4}{6},$$

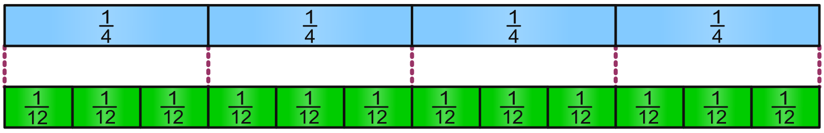


Example 2

If we divide each fourth into 3 equal parts, there are $4 \times 3 = 12$ equal parts on this fraction strip.

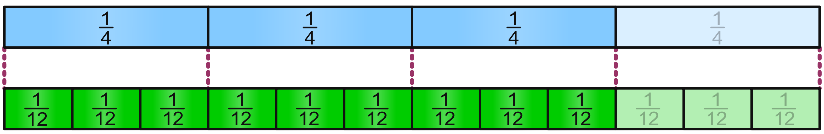


These are twelfths.



Since we divided each fourth into 3 equal parts, each of these parts is one third the size of $\mathbf{\frac{1}{4}}$.
So for a fraction with denominator 12 that is supposed to be the same size as a fraction with denominator 4, we need three times as many parts.
Therefore, if we multiply the numerator and the denominator of $\mathbf{\frac{3}{4}}$ by 3, we get an equivalent fraction, namely $\mathbf{\frac{9}{12}}$.
We have:

$$\frac{3}{4} = \frac{3 \times 3}{4 \times 3} = \frac{9}{12} \, .$$



What we saw above applies to all other fractions as well. The following statement holds for all natural numbers greater than 0.

> If both the numerator and the denominator of a fraction are multiplied by the same number, the result is a fraction of the same size.

We can also write this clearly using variables. If you want to make a general statement about a fraction in mathematics, you use variables — that is, letters that can stand for numbers. A general fraction is written, for example, as $\frac{\mathbf{a}}{\mathbf{b}}$. If it is to be multiplied by any natural number, that number is also written as a variable, for example **n**.

We can now write the statement above like this:

For any fraction $\frac{\mathbf{a}}{\mathbf{b}}$ and any natural number **n** (except 0), the following holds:

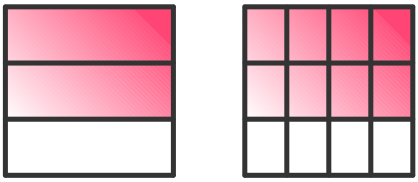
$$\frac{a}{b} = \frac{a \times n}{b \times n}$$

We call the process of multiplying both the numerator and the denominator of a fraction by the same number *creating an equivalent fraction*.

Visualizing Equivalent Fractions

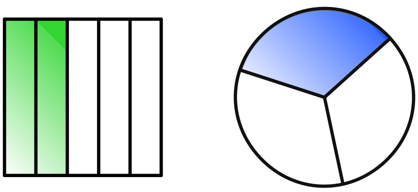
There are many ways to visualize the idea of creating equivalent fractions.

We can use a pizza to understand that the value of a fraction doesn't change when we divide each part into smaller equal parts: A pizza stays the same size, no matter whether it's divided into 2, 4, 6, 8, ... parts. So instead of eating one half of a pizza, we can also eat $\frac{2}{4}$, $\frac{3}{6}$, or $\frac{4}{8}$ of the pizza.



$\frac{2}{3}$ of the area of the left square is shaded red. The right square has an additional division of the area, but the amount of the area that is red has stayed the same. We have:

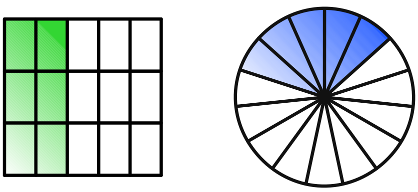
$$\frac{2}{3} = \frac{2 \times 4}{3 \times 4} = \frac{8}{12}$$



The areas of the square and the circle are the same. $\frac{2}{5}$ of the square is shaded green and $\frac{1}{3}$ of the circle is shaded blue. Which shaded area is larger?

We can divide each fifth of the square into 3 parts. Then the shaded area becomes:

$$\frac{2}{5} = \frac{2 \times 3}{5 \times 3} = \frac{6}{15}$$



We can also divide each third of the circle into 5 parts. Then the shaded area becomes:

$$\frac{1}{3} = \frac{1 \times 5}{3 \times 5} = \frac{5}{15}$$

Since $\frac{6}{15}$ is greater than $\frac{5}{15}$, the green area is the larger one.

If we have a satellite photo and want to estimate what portion of the area is covered by water, we can divide the photo into equal-sized units and count how many of the units show land and how many show water.
Dividing into sixteenths is probably too coarse to make this method useful, so we will have to subdivide the sixteenths into smaller units. Which number we should use to create equivalent fractions depends on how precise we want our estimate to be. We might also consider whether dividing into rectangles is the best solution, or whether dividing into squares might be more useful.