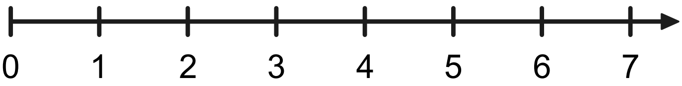
Fractions

Definitions

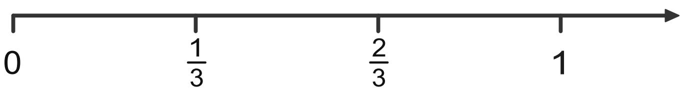
So far, we have worked with the numbers 0, 1, 2, 3, 4, 5, ... These numbers are called **natural numbers**, and we have represented them on the number line.



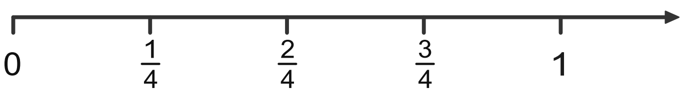
With natural numbers, we can count whole things like tomatoes, sunrises, or ideas. We can add, subtract, multiply, and divide these numbers.
A tomato can be divided into 4 equal parts. We can also perform arithmetic with these parts. If we combine two of these parts with two more, we get a whole tomato again. But it's not $2+2=1$.
So if we want to do arithmetic with parts of a whole, we need other numbers—numbers that lie on the number line between the natural numbers.

The number that lies exactly halfway between 0 and 1 is called: $\frac{1}{2}$ (read: one half).

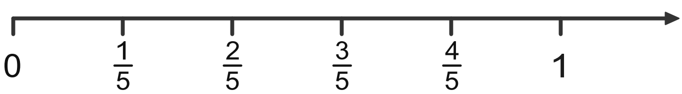
If we divide a whole—that is, the segment between 0 and 1—into three equal parts, we get the numbers $\frac{1}{3}$ (read: one third) and $\frac{2}{3}$ (read: two thirds).



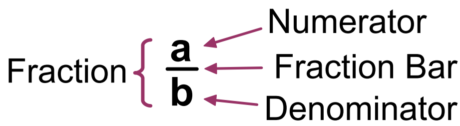
If we divide a whole into 4 equal parts, we get the numbers $\frac{1}{4}$ (read: one fourth), $\frac{2}{4}$ (read: two fourths), and $\frac{3}{4}$ (read: three fourths).



If we divide a whole into 5 equal parts, we get fifths.



In this way, we can create many more numbers. These numbers are called **fractions**. A **fraction** consists of a **fraction bar**, a whole number above the bar, and a whole number below the bar. The number above the bar is called the **numerator**, and the number below the bar is called the **denominator**.



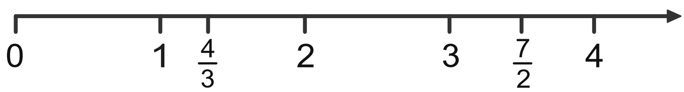
> **Definition**: Fractions are numbers made up of equal parts of a whole. The denominator of a fraction tells us *what kind* of parts the fraction is made of, and the numerator tells us *how many* of those parts it consists of.

A fraction is also called a **rational number**.
All fractions together make up the **set of rational numbers**. We denote this set with the symbol $\mathbb{Q}$.

Further Properties

1. Fractions Greater Than 1

Fractions can also be greater than 1. For example, the fractions $\frac{4}{3}$ and $\frac{7}{2}$ (read: seven halves) lie to the right of 1 on the number line.



2. Denominators Must Not Be Zero

We have defined fractions as numbers that arise when a whole is divided into equally sized parts. Since it is not possible to divide something into 0 parts, we stipulate that fractions with a denominator of 0 shall not exist. Even if we do not divide a whole at all, it still consists of *one* part — not of none at all.

3. The denominator can be equal to 1

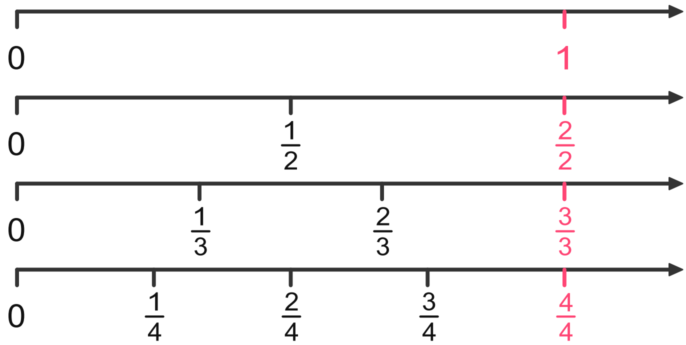
We define that the fraction $\frac{1}{1}$ (read: one whole) is equal to 1. Furthermore, $\frac{2}{1}$ (read: two wholes) is equal to 2, $\frac{3}{1}$ is equal to 3, and so on.

4. Multiple numbers at the same place on the number line

If we divide a whole into

- 2 equal parts, then 2 of those parts are as big as the whole.
- 3 equal parts, then 3 of those parts are as big as the whole.

- 4 equal parts, then 4 of those parts are as big as the whole.

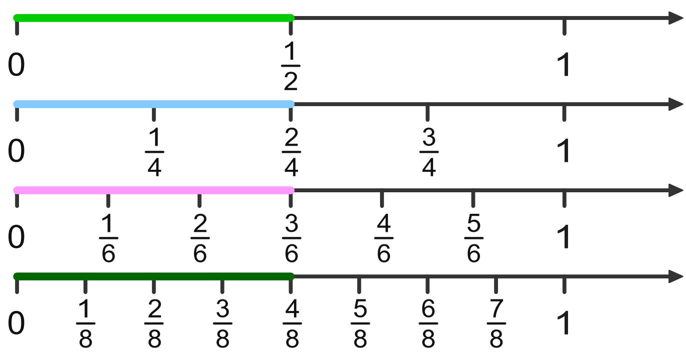


We therefore define: At the place on the number line where we find the 1, we also find other numbers. It holds that:

$$1 = \frac{1}{1} = \frac{2}{2} = \frac{3}{3} = \frac{4}{4} = \dots$$

There are also multiple numbers at other places on the number line.
If we divide a whole into

- 2 equal parts, then one of those parts is as big as $\frac{1}{2}$.
- 4 equal parts, then 2 of those parts are as big as $\frac{1}{2}$.
- 6 equal parts, then 3 of those parts are as big as $\frac{1}{2}$.
- 8 equal parts, then 4 of those parts are as big as $\frac{1}{2}$.



We see:

$$\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8} = \dots$$

The same applies to other fractions, such as:

$$\frac{1}{3} = \frac{2}{6} = \frac{3}{9} = \frac{4}{12} = \ldots \text{ and}$$

$$\frac{3}{7} = \frac{6}{14} = \frac{9}{21} = \frac{12}{28} = \ldots.$$

We will explain these relationships when we talk about how to extend fractions. But here's something we can already guess:

If the denominator of a fraction is twice the numerator, then the fraction equals $\mathbf{\frac{1}{2}}$.

If the denominator of a fraction is three times the numerator, then the fraction equals $\mathbf{\frac{1}{3}}$.

5. The Bigger the Denominator, the Smaller the Parts

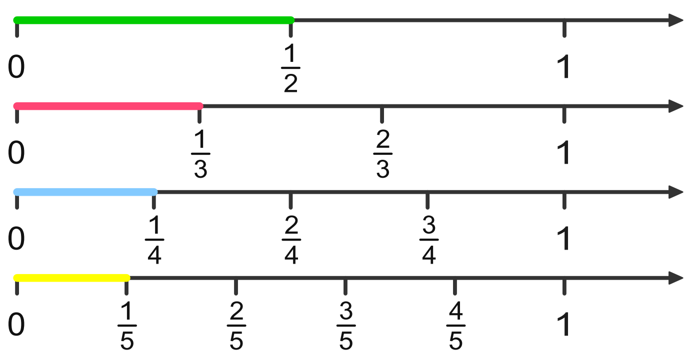
If we divide a whole into two equal parts, each of these parts is as big as $\mathbf{\frac{1}{2}}$. If we divide a whole into *more* than two equal parts, each of these parts is *smaller* than $\mathbf{\frac{1}{2}}$. Therefore, for example:

$\frac{1}{3}$ is smaller than $\frac{1}{2}$,

$\frac{1}{4}$ is smaller than $\frac{1}{2}$,

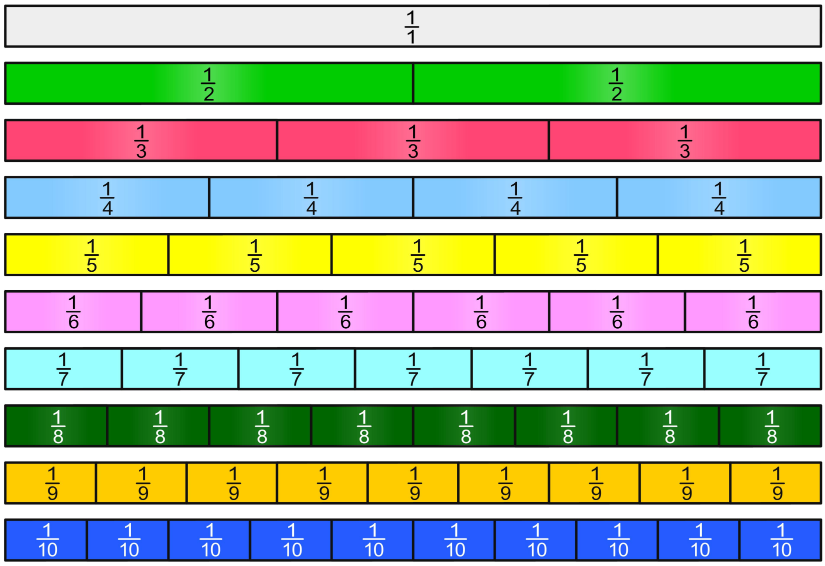
$\frac{1}{5}$ is smaller than $\frac{1}{2}$, and so on.

If we divide a whole into more than three equal parts, each of these parts is *smaller* than $\mathbf{\frac{1}{3}}$. For example, $\mathbf{\frac{1}{4}}$, $\mathbf{\frac{1}{5}}$, etc. are each smaller than $\mathbf{\frac{1}{3}}$.



Fraction Strips

To help us visualize fractions and their properties, we use fraction strips. Here are the fraction strips up to tenths.



We can place the fraction strips on the number line.