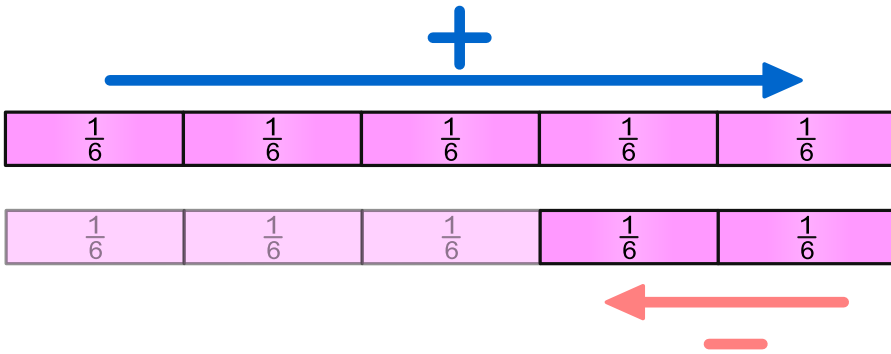# 1 Subtracting Fractions

1.1 Subtracting Like Fractions

To visualize the addition of fractions using fraction strips, we simply placed two fraction strips next to each other. To visualize the subtraction of fractions using fraction strips, we can think of the fractional parts as steps:

The expression $\frac{5}{6} - \frac{2}{6}$ then becomes:

First, we take 5 sixth-steps to the right and then 2 sixth-steps to the left. We end up at 3 sixth-steps, and that is the result of the subtraction.
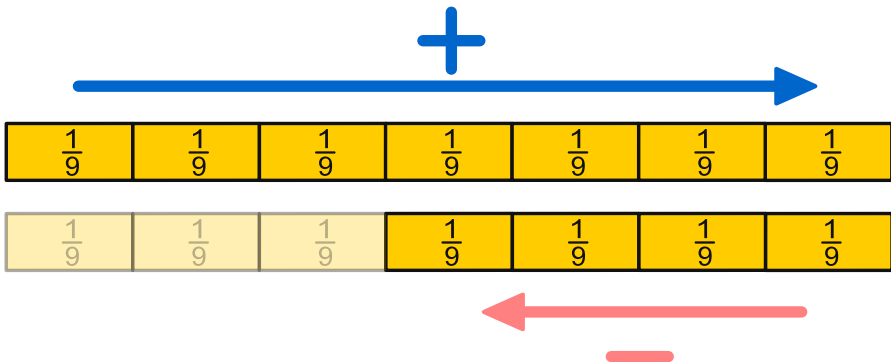
So: $\frac{5}{6} - \frac{2}{6} = \frac{3}{6}$.

When we added like fractions, we added the numerators and kept the common denominator. When we subtract like fractions, we proceed in a similar way: we subtract the numerators and keep the common denominator. The calculation looks like this:

$$\frac{5}{6} - \frac{2}{6} = \frac{5-2}{6} = \frac{3}{6}$$

Let's take a look at another example of subtraction:

Here we first take 7 steps of ninths to the right and then 4 steps of ninths to the left. We end up at 3 ninths.

We can also express this situation using numbers:

$$\frac{7}{9} - \frac{4}{9} = \frac{7-4}{9} = \frac{3}{9}$$

When subtracting fractions, we can proceed very similarly to how we add fractions.

> Two like fractions are subtracted
> by subtracting the numerators.
> The denominator stays the same.

We write this as a formula:

$$\frac{a}{c} - \frac{b}{c} = \frac{a-b}{c}$$

As usual, we call the result of a subtraction the *difference.*

We always write the difference as a simplified fraction. For example:

$$\frac{11}{8} - \frac{5}{8} = \frac{11-5}{8} = \frac{6}{8} = \frac{6 \div 2}{8 \div 2} = \frac{3}{4}$$

If the difference is a whole number, then we write the result as that whole number. For example:

$$\frac{15}{4} - \frac{3}{4} = \frac{15-3}{4} = \frac{12}{4} = \frac{12 \div 4}{4 \div 4} = \frac{3}{1} = 3$$

1.2   Subtracting Unlike Fractions

When we want to subtract unlike fractions, we first rewrite the fractions with a common denominator, and then subtract the like fractions. For example:
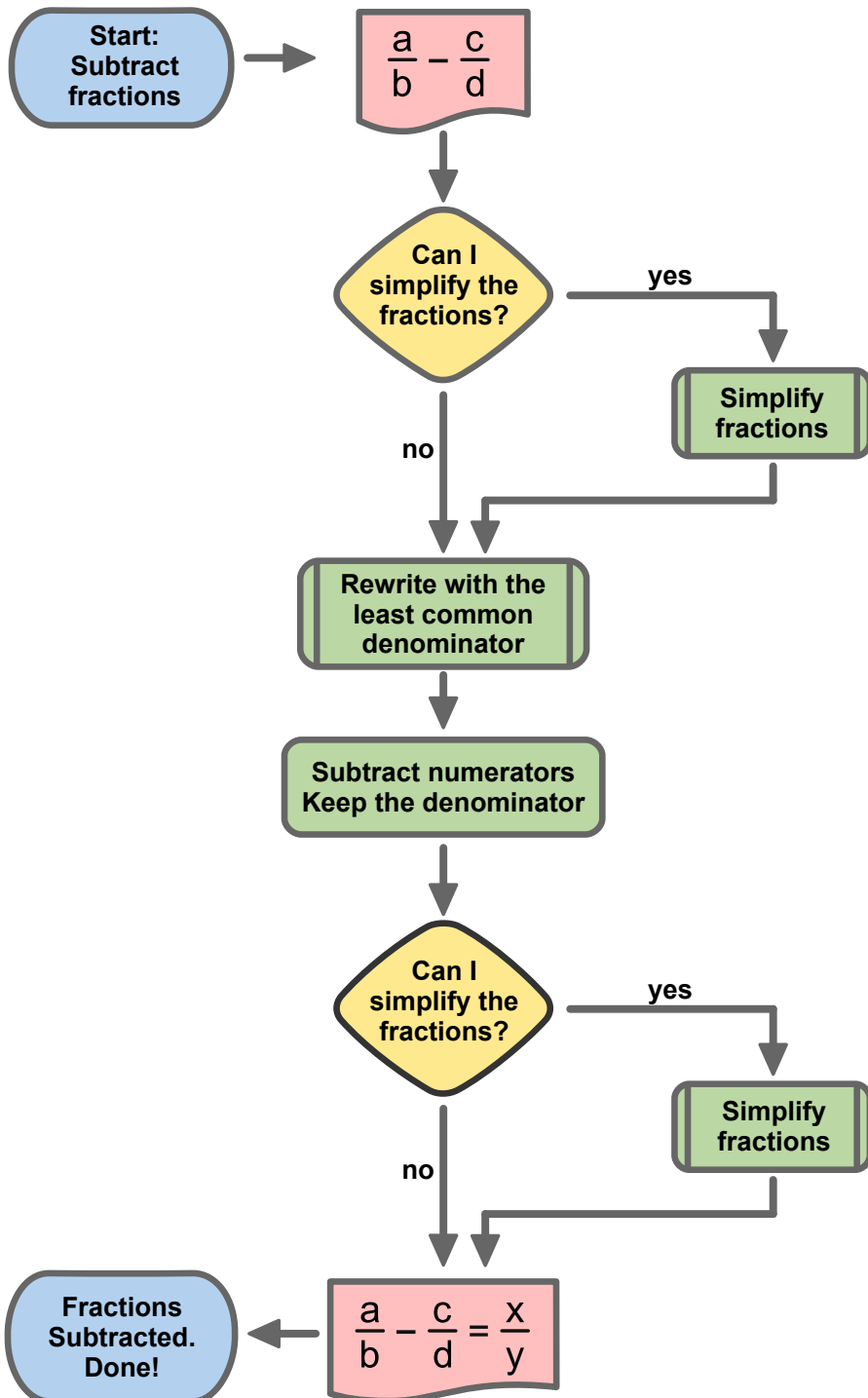
$$\frac{5}{8} - \frac{3}{20} = \frac{5 \times 5}{8 \times 5} - \frac{3 \times 2}{20 \times 2} = \frac{25}{40} - \frac{6}{40} = \frac{25-6}{40} = \frac{19}{40}$$

and

$$\frac{9}{10} - \frac{6}{7} = \frac{9 \times 7}{10 \times 7} - \frac{6 \times 10}{7 \times 10} = \frac{63}{70} - \frac{60}{70} = \frac{63-60}{70} = \frac{3}{70}$$

.

1.3 Flowchart

The flowchart shows the procedure we use to subtract fractions.

```
┌──────────────┐        ┌──────────────┐
│   Start:     │   →    │  a     c     │
│  Subtract    │        │  ─  −  ─     │
│  fractions   │        │  b     d     │
└──────────────┘        └──────────────┘
                              │
                              ▼
                        ◆ Can I           yes    ┌──────────────┐
                          simplify the  ───────→ │  Simplify    │
                          fractions? ◆           │  fractions   │
                              │                  └──────────────┘
                          no  │                        │
                              ▼                        │
                        ┌──────────────┐ ◄─────────────┘
                        │ Rewrite with the │
                        │ least common     │
                        │ denominator      │
                        └──────────────┘
                              │
                              ▼
                        ┌──────────────────┐
                        │ Subtract numerators │
                        │ Keep the denominator│
                        └──────────────────┘
                              │
                              ▼
                        ◆ Can I           yes    ┌──────────────┐
                          simplify the  ───────→ │  Simplify    │
                          fractions? ◆           │  fractions   │
                              │                  └──────────────┘
                          no  │                        │
┌──────────────┐              ▼                        │
│  Fractions   │        ┌──────────────────┐ ◄─────────┘
│  Subtracted. │ ◄───── │  a     c     x   │
│  Done!       │        │  ─  −  ─  =  ─   │
└──────────────┘        │  b     d     y   │
                        └──────────────────┘
```

1.4 Examples

Let's look at a few examples.

Example 1

1. Goal?

We want to subtract $\frac{1}{3}$ from $\frac{3}{4}$, which means we want to calculate $\frac{3}{4} - \frac{1}{3}$.

2. Simplify?

Neither $\frac{3}{4}$ nor $\frac{1}{3}$ can be simplified.

3. Least Common Denominator

$$\frac{3}{4} = \frac{3 \times 3}{4 \times 3} = \frac{9}{12} \quad , \quad \frac{1}{3} = \frac{1 \times 4}{3 \times 4} = \frac{4}{12}$$
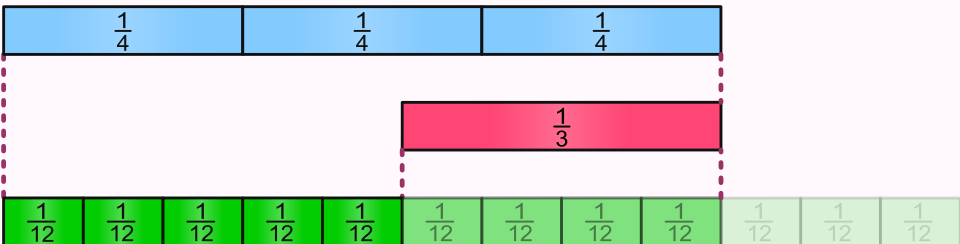
4. Subtract

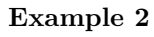$$\frac{9}{12} - \frac{4}{12} = \frac{5}{12}$$

5. Simplify?

$\frac{5}{12}$ cannot be simplified.

6. Final Result

$$\frac{3}{4} - \frac{1}{3} = \frac{3 \times 3}{4 \times 3} - \frac{1 \times 4}{3 \times 4} = \frac{9}{12} - \frac{4}{12} = \frac{5}{12}$$

Example 2

1. Goal?

We want to subtract $\frac{4}{6}$ from $\frac{14}{15}$, which means we want to calculate $\frac{14}{15} - \frac{4}{6}$.

2. Simplify?

$$\frac{14}{15} \quad \text{cannot be simplified;} \quad \frac{4}{6} = \frac{4 \div 2}{6 \div 2} = \frac{2}{3}$$

3. Least Common Denominator

$$\frac{14}{15} \quad , \quad \frac{2}{3} = \frac{2 \times 5}{3 \times 5} = \frac{10}{15}$$

4. Subtract

$$\frac{14}{15} - \frac{10}{15} = \frac{4}{15}$$

5. Simplify?

$$\frac{4}{15} \quad \text{cannot be simplified.}$$

6. Final Result

$$\frac{14}{15} - \frac{4}{6} = \frac{14}{15} - \frac{4 \div 2}{6 \div 2} = \frac{14}{15} - \frac{2}{3} = \frac{14}{15} - \frac{2 \times 5}{3 \times 5} = \frac{14}{15} - \frac{10}{15} = \frac{4}{15}$$

Example 3

1. Goal?

We want to subtract $\frac{13}{26}$ from $\frac{18}{21}$, which means we want to calculate $\frac{18}{21} - \frac{13}{26}$.

2. Simplify?

$$\frac{18}{21} = \frac{18 \div 3}{21 \div 3} = \frac{6}{7} \text{ and } \frac{13}{26} = \frac{13 \div 13}{26 \div 13} = \frac{1}{2}$$

3. Least Common Denominator

$$\frac{6}{7} = \frac{6 \times 2}{7 \times 2} = \frac{12}{14} \quad , \quad \frac{1}{2} = \frac{1 \times 7}{2 \times 7} = \frac{7}{14}$$

4. Subtract

$$\frac{12}{14} - \frac{7}{14} = \frac{5}{14}$$

5. Simplify?

$$\frac{5}{14} \text{ cannot be simplified.}$$

6. Result

$$\frac{18}{21} - \frac{13}{26} = \frac{18 \div 3}{21 \div 3} - \frac{13 \div 13}{26 \div 13} = \frac{6}{7} - \frac{1}{2} = \frac{6 \times 2}{7 \times 2} - \frac{1 \times 7}{2 \times 7} = \frac{12}{14} - \frac{7}{14} = \frac{5}{14}$$

Example 4

1. Goal?

We want to subtract $\frac{20}{75}$ from $\frac{22}{24}$, which means we want to calculate $\frac{22}{24} - \frac{20}{75}$.

2. Simplify?

$$\frac{22}{24} = \frac{22 \div 2}{24 \div 2} = \frac{11}{12} \quad \text{and} \quad \frac{20}{75} = \frac{20 \div 5}{75 \div 5} = \frac{4}{15}$$

3. Least Common Denominator

$$\frac{11}{12} = \frac{11 \times 5}{12 \times 5} = \frac{55}{60} \quad , \quad \frac{4}{15} = \frac{4 \times 4}{15 \times 4} = \frac{16}{60}$$
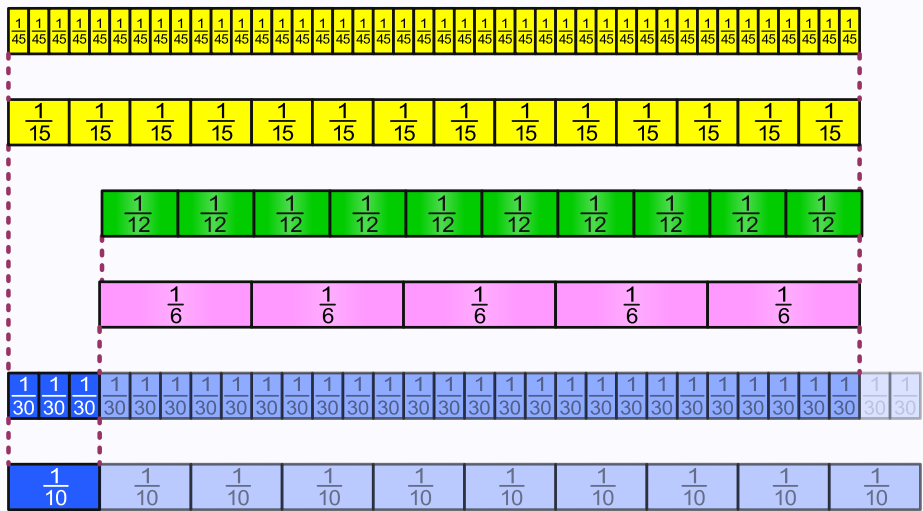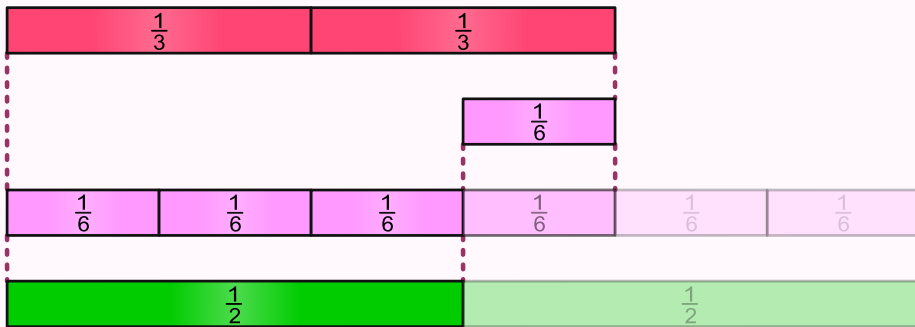
4. Subtract

$$\frac{55}{60} - \frac{16}{60} = \frac{39}{60}$$

5. Simplify?

$$\frac{39}{60} = \frac{39 \div 3}{60 \div 3} = \frac{13}{20}$$

6. Final Result

$$\frac{22}{24} - \frac{20}{75} = \frac{22 \div 2}{24 \div 2} - \frac{20 \div 5}{75 \div 5} = \frac{11}{12} - \frac{4}{15} = \frac{11 \times 5}{12 \times 5} - \frac{4 \times 4}{15 \times 4} = \frac{55}{60} - \frac{16}{60} = \frac{39}{60} = \frac{39 \div 3}{60 \div 3} = \frac{13}{20}$$

| $\frac{1}{24}$ | $\frac{1}{24}$ | $\frac{1}{24}$ | $\frac{1}{24}$ | $\frac{1}{24}$ | $\frac{1}{24}$ | $\frac{1}{24}$ | $\frac{1}{24}$ | $\frac{1}{24}$ | $\frac{1}{24}$ | $\frac{1}{24}$ | $\frac{1}{24}$ | $\frac{1}{24}$ | $\frac{1}{24}$ | $\frac{1}{24}$ | $\frac{1}{24}$ | $\frac{1}{24}$ | $\frac{1}{24}$ | $\frac{1}{24}$ | $\frac{1}{24}$ | $\frac{1}{24}$ | $\frac{1}{24}$ |

| $\frac{1}{12}$ | $\frac{1}{12}$ | $\frac{1}{12}$ | $\frac{1}{12}$ | $\frac{1}{12}$ | $\frac{1}{12}$ | $\frac{1}{12}$ | $\frac{1}{12}$ | $\frac{1}{12}$ | $\frac{1}{12}$ | $\frac{1}{12}$ |

| $\frac{1}{15}$ | $\frac{1}{15}$ | $\frac{1}{15}$ | $\frac{1}{15}$ |

| $\frac{1}{20}$ | $\frac{1}{20}$ | $\frac{1}{20}$ | $\frac{1}{20}$ | $\frac{1}{20}$ | $\frac{1}{20}$ | $\frac{1}{20}$ | $\frac{1}{20}$ | $\frac{1}{20}$ | $\frac{1}{20}$ | $\frac{1}{20}$ | $\frac{1}{20}$ | $\frac{1}{20}$ | $\frac{1}{20}$ | $\frac{1}{20}$ | $\frac{1}{20}$ | $\frac{1}{20}$ | $\frac{1}{20}$ | $\frac{1}{20}$ | $\frac{1}{20}$ |
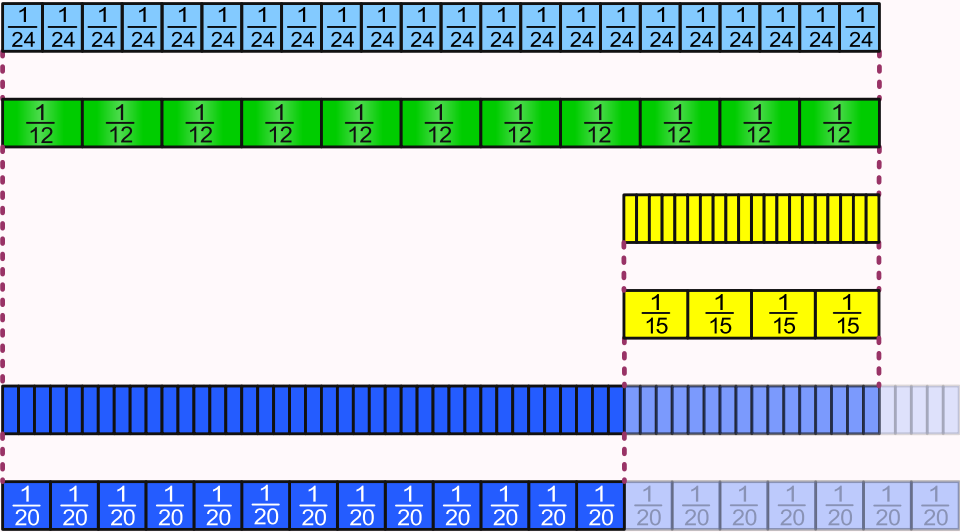
Example 5

1. Goal?

We want to subtract $\frac{10}{12}$ from $\frac{42}{45}$, so we calculate $\frac{42}{45} - \frac{10}{12}$.

2. Simplify?

$$\frac{42}{45} = \frac{42 \div 3}{45 \div 3} = \frac{14}{15} \text{ and } \frac{10}{12} = \frac{10 \div 2}{12 \div 2} = \frac{5}{6}$$

3. Least Common Denominator

$$\frac{14}{15} = \frac{14 \times 2}{15 \times 2} = \frac{28}{30} \quad , \quad \frac{5}{6} = \frac{5 \times 5}{6 \times 5} = \frac{25}{30}$$

4. Subtract

$$\frac{28}{30} - \frac{25}{30} = \frac{3}{30}$$

5. Simplify?

$$\frac{3}{30} = \frac{3 \div 3}{30 \div 3} = \frac{1}{10}$$

6. Final Result

$$\frac{42}{45} - \frac{10}{12} = \frac{42 \div 3}{45 \div 3} - \frac{10 \div 2}{12 \div 2} = \frac{14}{15} - \frac{5}{6} = \frac{14 \times 2}{15 \times 2} - \frac{5 \times 5}{6 \times 5} = \frac{28}{30} - \frac{25}{30} = \frac{3}{30} = \frac{3 \div 3}{30 \div 3} = \frac{1}{10}$$

1.5 Exercises

Exercise 1

1. Goal?

We want to subtract $\frac{1}{6}$ from $\frac{2}{3}$, so we calculate $\frac{2}{3} - \frac{1}{6}$.

2. Simplify?

Neither $\frac{2}{3}$ nor $\frac{1}{6}$ can be simplified.

3. Least Common Denominator

$$\frac{2}{3} = \frac{2 \times 2}{3 \times 2} = \frac{4}{6} \quad , \quad \frac{1}{6}$$

4. Subtract

$$\frac{4}{6} - \frac{1}{6} = \frac{3}{6}$$

5. Simplify?

$$\frac{3}{6} = \frac{3 \div 3}{6 \div 3} = \frac{1}{2}$$

6. Final Result

$$\frac{2}{3} - \frac{1}{6} = \frac{2 \times 2}{3 \times 2} - \frac{1}{6} = \frac{4}{6} - \frac{1}{6} = \frac{3}{6} = \frac{3 \div 3}{6 \div 3} = \frac{1}{2}$$

Exercise 2

1. Goal

We want to subtract $\frac{10}{34}$ from $\frac{15}{17}$, so we calculate $\frac{15}{17} - \frac{10}{34}$.

2. Simplify?

$\frac{15}{17}$ cannot be simplified; $\frac{10}{34} = \frac{10 \div 2}{34 \div 2} = \frac{5}{17}$

3. Least common denominator

$$\frac{15}{17} \quad , \quad \frac{5}{17}$$

4. Subtract

$$\frac{15}{17} - \frac{5}{17} = \frac{10}{17}$$

5. Simplify?

$\frac{10}{17}$ cannot be simplified.

6. Final Result

$$\frac{15}{17} - \frac{10}{34} = \frac{15}{17} - \frac{10 \div 2}{34 \div 2} = \frac{15}{17} - \frac{5}{17} = \frac{10}{17}$$
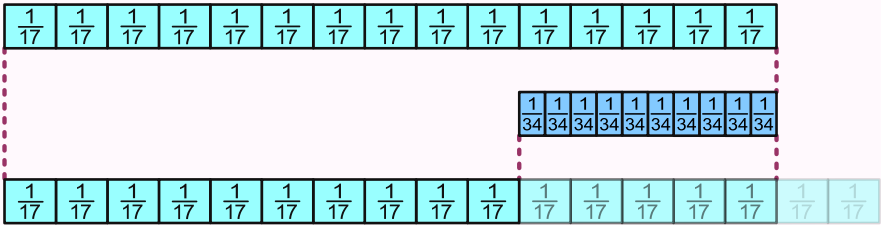
Exercise 3

1. Goal

We want to subtract $\frac{11}{20}$ from $\frac{22}{24}$, so we calculate $\frac{22}{24} - \frac{11}{20}$.

2. Simplify?

$$\frac{22}{24} = \frac{22 \div 2}{24 \div 2} = \frac{11}{12} \;\; ; \;\; \frac{11}{20}$$

3. Least common denominator

$$\frac{11}{12} = \frac{11 \times 5}{12 \times 5} = \frac{55}{60} \;\;,\;\; \frac{11}{20} = \frac{11 \times 3}{20 \times 3} = \frac{33}{60}$$
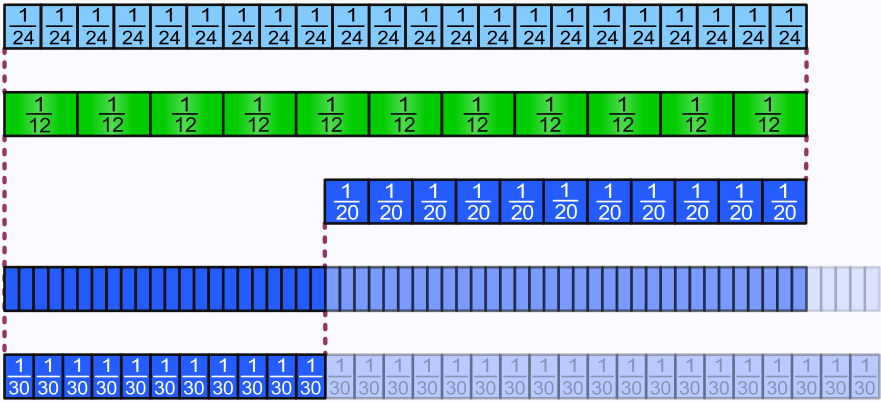
4. Subtract

$$\frac{55}{60} - \frac{33}{60} = \frac{22}{60}$$

5. Simplify?

$$\frac{22}{60} = \frac{22 \div 2}{60 \div 2} = \frac{11}{30}$$

6. Final Result

$$\frac{22}{24} - \frac{11}{20} = \frac{22 \div 2}{24 \div 2} - \frac{11}{20} = \frac{11}{12} - \frac{11}{20} = \frac{11 \times 5}{12 \times 5} - \frac{11 \times 3}{20 \times 3} = \frac{55}{60} - \frac{33}{60} = \frac{22}{60} = \frac{22 \div 2}{60 \div 2} = \frac{11}{30}$$

Exercise 4

1. Goal

We want to subtract $\frac{20}{56}$ from $\frac{25}{30}$, so we calculate $\frac{25}{30} - \frac{20}{56}$.

2. Simplify?

$$\frac{25}{30} = \frac{25 \div 5}{30 \div 5} = \frac{5}{6} \text{ and } \frac{20}{56} = \frac{20 \div 4}{56 \div 4} = \frac{5}{14}$$

3. Least common denominator

$$\frac{5}{6} = \frac{5 \times 7}{6 \times 7} = \frac{35}{42} \quad , \quad \frac{5}{14} = \frac{5 \times 3}{14 \times 3} = \frac{15}{42}$$
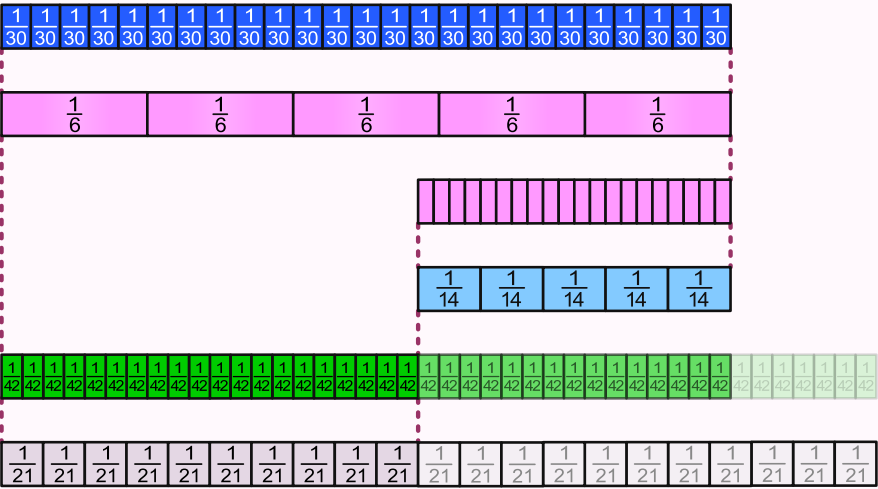
4. Subtract

$$\frac{35}{42} - \frac{15}{42} = \frac{20}{42}$$

5. Simplify?

$$\frac{20}{42} = \frac{20 \div 2}{42 \div 2} = \frac{10}{21}$$

6. Final Result

$$\frac{25}{30} - \frac{20}{56} = \frac{25 \div 5}{30 \div 5} - \frac{20 \div 4}{56 \div 4} = \frac{5}{6} - \frac{5}{14} = \frac{5 \times 7}{6 \times 7} - \frac{5 \times 3}{14 \times 3} = \frac{35}{42} - \frac{15}{42} = \frac{20}{42} = \frac{20 \div 2}{42 \div 2} = \frac{10}{21}$$

Exercise 5

1. Goal

We want to subtract $\frac{2}{15}$ from $\frac{15}{50}$, so we calculate $\frac{15}{50} - \frac{2}{15}$.

2. Simplify?

$$\frac{15}{50} = \frac{15 \div 5}{50 \div 5} = \frac{3}{10} \; ; \; \frac{2}{15}$$

3. Least Common Denominator

$$\frac{3}{10} = \frac{3 \times 3}{10 \times 3} = \frac{9}{30} \;,\; \frac{2}{15} = \frac{2 \times 2}{15 \times 2} = \frac{4}{30}$$
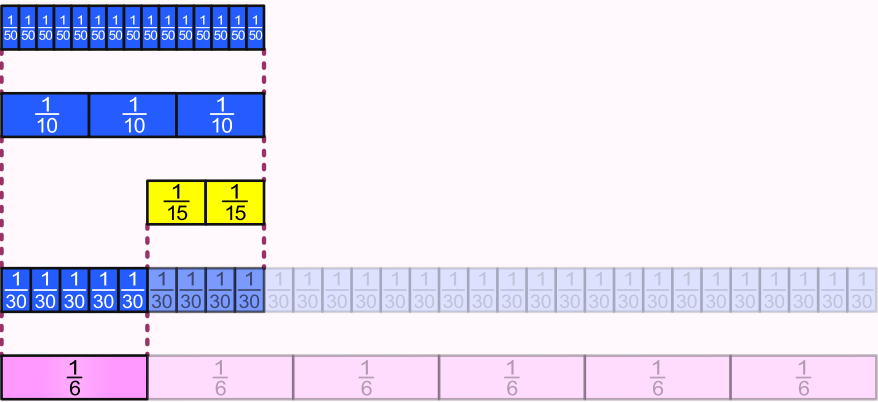
4. Subtract

$$\frac{9}{30} - \frac{4}{30} = \frac{5}{30}$$

5. Simplify?

$$\frac{5}{30} = \frac{5 \div 5}{30 \div 5} = \frac{1}{6}$$

6. Final Result

$$\frac{15}{50} - \frac{2}{15} = \frac{15 \div 5}{50 \div 5} - \frac{2}{15} = \frac{3}{10} - \frac{2}{15} = \frac{3 \times 3}{10 \times 3} - \frac{2 \times 2}{15 \times 2} = \frac{9}{30} - \frac{4}{30} = \frac{5}{30} = \frac{5 \div 5}{30 \div 5} = \frac{1}{6}$$

Exercise 6

1. Goal

We want to subtract $\frac{39}{63}$ from $\frac{65}{60}$, so we calculate $\frac{65}{60} - \frac{39}{63}$.

2. Simplify?

$$\frac{65}{60} = \frac{65 \div 5}{60 \div 5} = \frac{13}{12} \; ; \; \frac{39}{63} = \frac{39 \div 3}{63 \div 3} = \frac{13}{21}$$

3. Least Common Denominator

$$\frac{13}{12} = \frac{13 \times 7}{12 \times 7} = \frac{91}{84} \; , \; \frac{13}{21} = \frac{13 \times 4}{21 \times 4} = \frac{52}{84}$$

4. Subtract

$$\frac{91}{84} - \frac{52}{84} = \frac{39}{84}$$

5. Simplify?

$$\frac{39}{84} = \frac{39 \div 3}{84 \div 3} = \frac{13}{28}$$

6. Final Result

$$\frac{65}{60} - \frac{39}{63} = \frac{65 \div 5}{60 \div 5} - \frac{39 \div 3}{63 \div 3} = \frac{13}{12} - \frac{13}{21} = \frac{13 \times 7}{12 \times 7} - \frac{13 \times 4}{21 \times 4} = \frac{91}{84} - \frac{52}{84} = \frac{39}{84} = \frac{39 \div 3}{84 \div 3} = \frac{13}{28}$$

Exercise 7

1. Goal?

We want to subtract $\frac{3}{7}$ from $\frac{3}{4}$, so we calculate $\frac{3}{4} - \frac{3}{7}$.

2. Simplify?

Neither $\frac{3}{4}$ nor $\frac{3}{7}$ can be simplified.

3. Least common denominator

$$\frac{3}{4} = \frac{3 \times 7}{4 \times 7} = \frac{21}{28} \quad , \quad \frac{3}{7} = \frac{3 \times 4}{7 \times 4} = \frac{12}{28}$$

4. Subtract

$$\frac{21}{28} - \frac{12}{28} = \frac{9}{28}$$

5. Simplify? $\frac{9}{28}$ cannot be simplified.

6. Final Result

$$\frac{3}{4} - \frac{3}{7} = \frac{3 \times 7}{4 \times 7} - \frac{3 \times 4}{7 \times 4} = \frac{21}{28} - \frac{12}{28} = \frac{9}{28}$$