Why is 2 to the power of 0 equal to 1?

Powers are a tricky thing in mathematics. It starts off very simply: For example, $2^3 = 2 \times 2 \times 2 = 8$, because the "exponent" tells us how many times the number 2 should be multiplied. Accordingly, $2^5 = 2 \times 2 \times 2 \times 2 \times 2 = 32$ and $2^2 = 2 \times 2 = 4$. But when we ask what 2^1 or 2^0 could be, things suddenly get a lot more complicated. If we look at the fact that when we write out 2^3 we have written the factor 2 three times, and for 2^2 we have written it twice, it would make a certain amount of sense if 2^1 were equal to 2. However, 2 is no longer a factor if it just stands there by itself. And what should we do with 2^0? Write nothing at all?

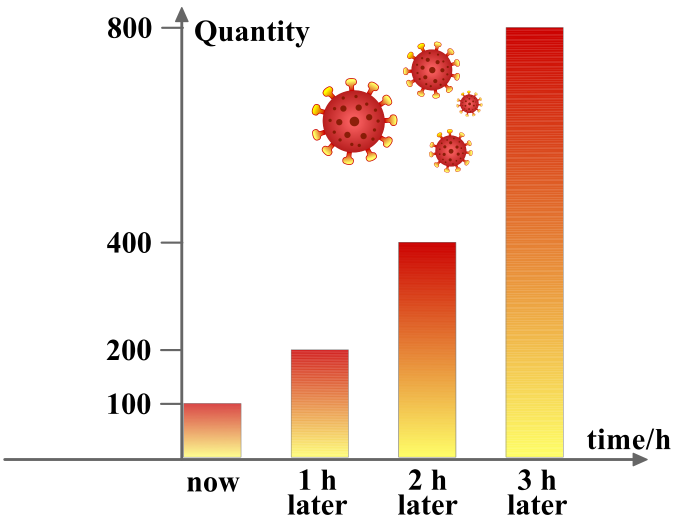

Fig. 1 Numbers of viruses

To arrive at meaningful results, we can imagine the growth of viruses (see Fig. 1). Suppose there are 100 viruses somewhere, and they reproduce in such a way that their number doubles every hour. Then there are 100 viruses now, after one hour there are 200, after two hours 400, and after three hours 800 viruses.
We calculate the number of viruses after one hour by multiplying the current number of viruses by 2. The number after two hours is obtained by multiplying by 2^2, and the number after three hours by multiplying by 2^3 (Fig. 2).

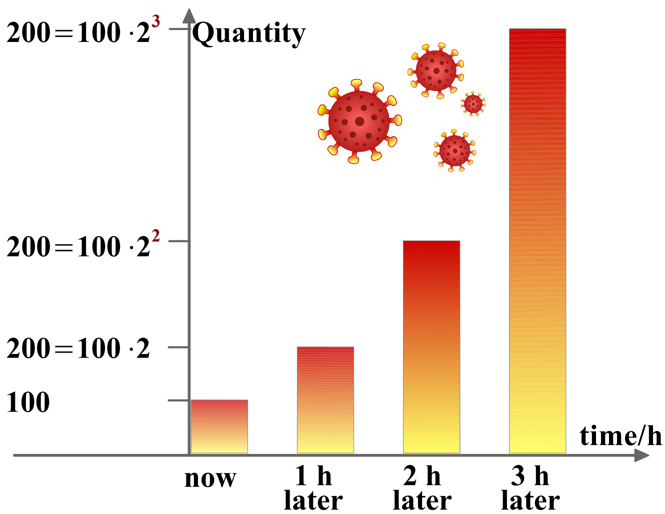

Fig. 2 Calculating the numbers

If the exponents in the multiplications with 2^2 and 2^3 tell us how many times we multiplied by 2, it makes sense to write $200 = 100 \times 2^1$, since we multiply 100 by 2 once to get 200.
How many times do we have to multiply 100 by 2 in order to get 100 as a result? Correct: not at all! So $100 = 100 \times 2^0$. But if multiplying 100 by 2^0 should not change the result 100, then it must hold that:

$$2^0 = 1$$

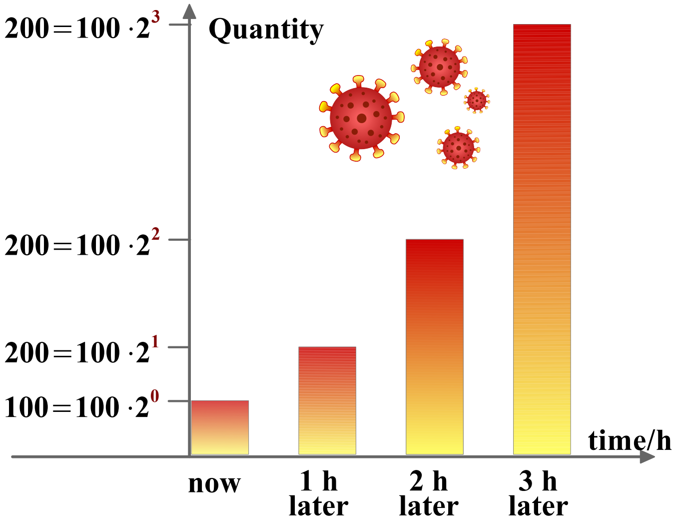

Fig. 3 $2^0 = 1$

One might wonder whether you can just define something like that in mathematics. Well, in principle, yes. Whether a given definition makes sense, however, is another question. The following definitions, for instance, will probably not catch on.

Definition 1 $2 + 2 = 5$

Definition 2 $a \times (b + c) = a \times b + c$

Definition 3 $\dfrac{a}{b} + \dfrac{c}{d} = \dfrac{a + c}{b + c}$

On the other hand, mathematical progress usually comes through new definitions. For instance, Definition 3 could serve as the basis of a new kind of fraction arithmetic. Whether this new fraction arithmetic would be in any way better than the traditional one is something its supporters would probably have to explain.

Now, what about our definition $2^0 = 1$? Is this definition reasonable?
If we define, with this definition (along with a few others), the function

$$f(x) = 100 \times 2^x$$

we get the graph shown in Fig. 4. This graph describes the growth of the viruses for every possible point in time. Any other definition of 2^0 would break the graph. That's quite a strong argument!

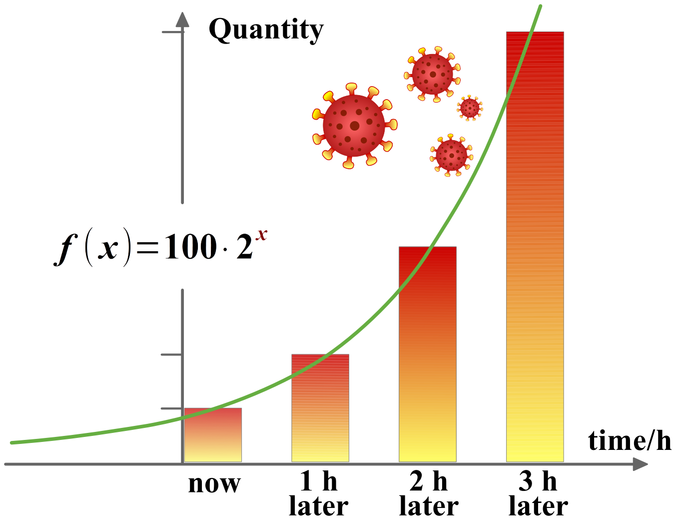

Fig. 4 Growth function