Why Does a Negative times a Negative Equal a Positive?

There are many ways to imagine $-(-2)$. Here's one with chocolate cookies: 9 cookies are available.

Fig. 1 9 chocolate cookies (Cookie by Corleto/Unsplash)

I'm allowed to eat 5 of them.

Fig. 2 9 - 5 chocolate cookies

Now I eat 2 fewer than I'm allowed to.

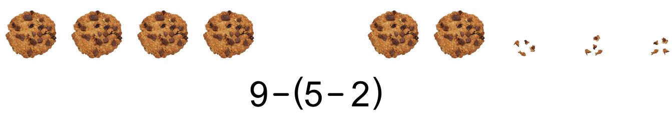

Fig. 3 $9 - (5 - 2)$ chocolate cookies

So I put 2 cookies back.

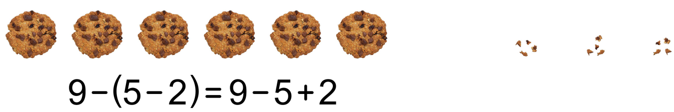

Fig. 4 9 - 5 + 2 chocolate cookies

Have we actually shown that $-(-2) = +2$? We could have written it a bit more clearly:

$$9 - (5 - 2) = 9 - 5 - (-2) = 9 - 5 + 2$$

As we can see, $-(-2)$ has turned into the number $+2$.

We can make it even clearer if we recall how to work with parentheses. Here's the distributive property:

$$a \times (b + c) = a \times b + a \times c$$

To get the same calculation as with the cookies, we substitute -1 for a, 5 for b, and -2 for c. So

$$a \cdot (b + c) = a \cdot b + a \cdot c$$

$$-1 \cdot (5 + (-2)) = -1 \cdot 5 + (-1) \cdot (-2)$$

Fig. 5 Distributive property with negative numbers

And as we've seen in the example, $(-1) \times (-2)$ must now be equal to $+2$.

Not everyone finds this explanation sufficient, which might be because mathematics usually doesn't have to follow the rules of eating chocolate cookies.

That's why there are standard models we can use for understanding. One such model is the number line. Using a standard model means this: we want to assume that our calculations make sense if they make sense in the standard model. In our case, this means that if we can show on the number line that $9-(5-2) = 9-5+2$ makes sense (and we can show the same for other numbers), then we want to assume that "a negative times a negative equals a positive." So:

On the number line, we can think of the number 3 as an arrow pointing from left to right.

Fig. 6 We see a 3

We can imagine the calculation $5-2$ like this: an arrow of length 2, pointing in the opposite direction, is attached to an arrow of length 5 that points from left to right.

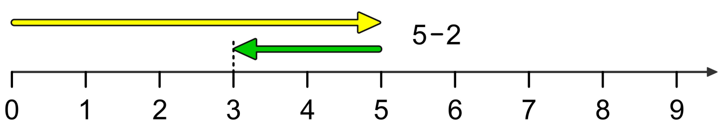

Fig. 7 $5-2$

As we know, $5-2=3$. That's why, in almost all formulas, equations, and expressions, we can replace 3 with $5-2$. So instead of adding 3, we can add $5-2$, or instead of subtracting 3, we can subtract $5-2$. Specifically, this means: if we calculate $9-3$, we turn the 3-arrow around and attach it to the 9-arrow.

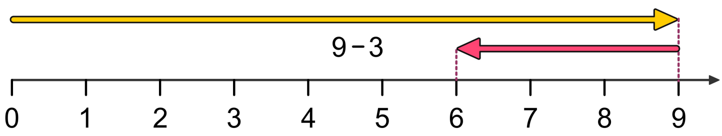

Fig. 8 $9-3$

Instead of subtracting 3 from 9, we can subtract $5-2$ from 9. To do that, we turn both arrows of $5-2$ around and attach them to the 9-arrow.

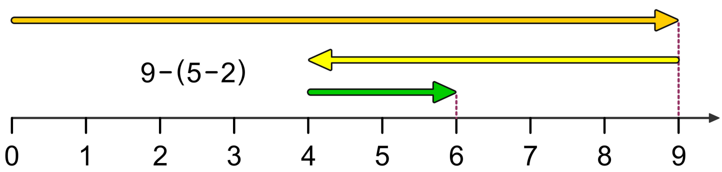

Fig. 9 $9-(5-2)$

Just as it looks on the number line, we get the same result when we calculate $9-5+2$. To do that, we first attach the reversed 5-arrow to the 9-arrow and then attach the positive 2-arrow to this result.

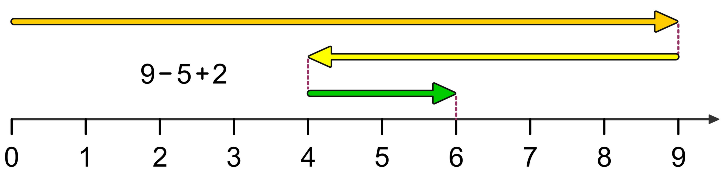

Fig. 10 $9-5+2$

So we get: $9-(5-2) = 9-5+2$

So if we can agree that a negative times a negative equals a positive when it works in this standard model, then we have at least shown that here with one example. But that's also what we're used to in everyday life: instead of taking three steps to the left, we can also first take five steps to the left and then two steps to the right to end up at the same point.